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# The Optimal Dole with Risk Aversion and Job Destruction

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January 1997

Revised February 1998

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## ABSTRACT

This paper extends earlier research on optimal unemployment insurance (UI) by developing an equilibrium search model that encompasses simultaneously several theoretical and institutional features that have been treated one-by-one (or not at all) in previous discussions of optimal UI. In particular, the model we develop allows us to determine the optimal potential duration of UI benefits as well as the optimal UI benefit amount; assumes (realistically) that not all workers are eligible for UI benefits; allows examination of various degrees of risk aversion by workers; models labor demand so that the job destruction effects of UI are taken into account; and treats workers as heterogeneous. The model suggests that the current statutory replacement rate of 50 percent provided by most states in the United States is close to optimal, but that the current potential duration of benefits (which is usually 26 weeks) is probably too short. This basic result—that the optimal UI system is characterized by a fairly low replacement rate and a long potential duration—conflicts with most of the existing literature on optimal UI. We argue, however, that the result is consistent with a large literature on optimal insurance contracts in the presence of moral hazard.

## 1. INTRODUCTION

In 1993, expenditures in the United States on unemployment insurance (UI) amounted to about .75 percent of GDP. In Canada, France, Germany, and the United Kingdom, expenditures on UI were about 2 percent of GDP; in Japan, just .3 percent of GDP (OECD 1995, Table T). These differences reflect differences in both labor markets and in the generosity of UI across the countries. But, in all cases, the primary intent of these expenditures was to provide a safety net for workers facing employment risk who were unable to self-insure by saving enough while employed to smooth consumption across jobless spells. Since these workers are unable to purchase UI in the private sector due to problems associated with moral hazard and adverse selection, the employment insurance is provided publicly. But, governments cannot monitor perfectly the effort put forth by the unemployed to find new jobs, and by providing benefits to the unemployed, the government reduces the incentives workers face to seek reemployment. Thus, there is a tradeoff—if too much public insurance is provided, the unemployed will not work hard enough to find new jobs, but, if too little insurance is provided, the unemployed will bear too much employment risk. In devising an optimal UI program, the government must find a way to provide adequate insurance without substantially reducing the incentive to seek new jobs.

Existing UI programs are quite diverse in the benefits they provide. In the United States, UI is a collection of state-federal programs that differ in the replacement rate paid to most workers and in the potential duration of benefits. Nevertheless, the most common characterization of the current U.S. UI program is that it provides a benefit equal to roughly 50 percent of the wage earned on the previous job for one-half of a year after a worker loses her job.<sup>1</sup> Similar programs are in place in

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<sup>1</sup> There is a cap on benefits which reduces the replacement rate below 50 percent for most high-wage workers.

Canada, the United Kingdom, and most other developed countries. In many cases the benefit rate is higher and in most cases the potential duration of benefits is longer.

Recently a large literature has developed that investigates whether existing UI programs are optimal in a variety of senses. Some papers focus on benefit rates (are they too high or too low?), some focus on the time path of benefits (should benefits be constant, rise or fall over the spell of unemployment?), some focus on the welfare effects of these programs (what is the deadweight loss associated with current programs when compared with optimal programs or no program at all?), and some focus on the potential duration of benefits (should benefits be offered for shorter or longer time periods?). For the most part, the prevailing view offered by most of these papers is that current programs are poorly designed and overly generous.

A problem with using the existing literature to draw policy conclusions is that virtually all of the articles on optimal UI suffer from at least one significant shortcoming. For example, most of the studies that attempt to measure the welfare loss from current UI programs assume that workers are risk neutral, so that there can be no welfare gain from the insurance provided by the government. Papers that address the adequacy of current benefit rates often make the unrealistic assumption that the potential duration of benefits is infinite, leading the authors to draw conclusions that may be misleading. Articles that have investigated the optimal time path of benefits usually do not address the issue of program adequacy.

In this paper we offer some new results on optimal unemployment insurance programs when the benefit rate is constant over the spell of unemployment. These new results can be viewed as an extension of our earlier work, Davidson and Woodbury (1997), but they also provide some insights concerning the robustness of the findings of the literature mentioned above.

We begin in Section 2 by offering a brief description of the approach that is usually adopted in this literature. We then offer a critical review of several important contributions in the optimal UI literature. In Section 3 we introduce an extended version of the model we developed Davidson and Woodbury (1997). In Section 4 we present our results and compare them with those in the existing literature. This comparison allows us to illustrate the frailty of some previous results— minor changes in assumptions often drastically alter conclusions. This also allows us to show that the key parameter in determining whether current programs are too generous is the degree of worker risk aversion. For example, we find that if the degree of relative risk aversion is small (less than  $1/2$ ), then current UI programs in the United States are too generous. If, however, the degree of relative risk aversion is large (greater than  $1/2$ ), then current UI programs in the United States are not generous enough.

## 2. OPTIMAL UI: STRENGTHS AND WEAKNESSES OF PREVIOUS RESEARCH

Almost all of the papers in the optimal UI literature adopt search models of the labor market in which unemployed workers choose search effort to maximize expected utility. More generous UI increases the insurance offered to the unemployed, but also lowers optimal search effort, thereby triggering an increase in unemployment. Although most papers adopt a similar approach, they often differ in the questions that are addressed, the complexity of the models, and the assumptions that are used to simplify the analysis. In this section, we restrict attention to those papers that assume that the benefit rate is constant over the spell of unemployment. We use this assumption in our subsequent analysis, and it is consistent with most actual UI programs.<sup>2</sup>

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<sup>2</sup>Articles that consider the optimal path of benefits over the unemployment spell include Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). These papers address the following question: given that the government is going

The two most heavily cited papers on optimal unemployment insurance appeared in the same 1978 issue of the *Journal of Public Economics*. These papers were written by Martin N. Baily and J.S. Flemming and were so similar in approach and conclusions that they carried almost identical titles. Both authors use a search model of the labor market in which unemployed workers choose search effort to maximize expected lifetime utility. Workers are risk averse, so that insurance is desired, and an equilibrium model is used in order to capture the impact of UI on unemployment. However, neither author explicitly models firm behavior so that neither is able to capture the impact of UI on the number of job opportunities available to workers. This implies that all of the increase in unemployment from UI is due to its impact on search effort.

The papers differ in the time horizons considered (Baily uses a two-period model while Flemming uses an infinite horizon approach), the manner in which the capital market (and thus, saving) is handled, and the utility function used. Nevertheless, as we discuss below, they derive remarkably similar results.

Both authors have the same goal—to determine the optimal replacement rate assuming that the rate remains constant over the spell of unemployment. The results are then compared to replacement rates offered in the U.S. and the U.K. to determine whether or not current UI programs are too generous. Briefly, Baily and Flemming both find that if agents cannot save the optimal replacement rate lies in the 60 to 70 percent range. This result is robust, since it does not depend on the time horizon or the manner in which the authors calibrate their models. There is one exception—the result does depend on the degree of risk aversion. Baily assumes that the Arrow-

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to spend a fixed amount of money on unemployment compensation, how should the benefits be paid out to the unemployed (i.e., how should the benefits vary over the spell of unemployment)? The optimal size of the program is an open question in these papers.

Pratt measure of relative risk aversion is constant and equal to one, while Flemming assumes that the Arrow-Pratt measure of absolute risk aversion is constant and equal to one. For lower measures of risk aversion, they find lower optimal replacement rates.

When agents can save but capital markets are imperfect (so that workers can only partially self-insure), Baily and Flemming find that the optimal replacement rate falls by about 25-30 percentage points. Thus, they conclude that the optimal replacement rate is below 50 percent and that the current U.S. unemployment insurance program is too generous. Similar conclusions have been reached by Gruber (1994) who recently used Baily's framework to investigate benefit adequacy in the U.S.

In our earlier work, Davidson and Woodbury (1997), we extended the work of Baily and Flemming by dropping two of the assumptions that they used in their analysis—that all unemployed agents are eligible for UI benefits and that they receive such benefits for as long as they remain unemployed. In fact, less than 50 percent of the unemployed are eligible for UI benefits in the U.S. (Blank and Card 1991) while in the U.K. roughly 70 percent of the unemployed are eligible (Layard et al 1991). In addition, benefits are usually offered for only 26 weeks in most states of the U.S. and are limited in almost every other country. In section 4, we review our earlier results, which indicate that the conclusions reached by Bailey and Flemming are extremely fragile with respect to these two assumptions. We then go on to extend the Baily and Flemming analysis even further by explicitly modeling firm behavior and making the wage rate and the number of active firms endogenous. This allows us to capture the impact of UI on aggregate job opportunities and to see exactly how this alters our results.



In addition to Baily and Flemming, several macroeconomists have recently begun to criticize the generosity of current labor market policies. For example, some have argued that the disincentive effects of UI are so strong that they have led to a significant increase in the unemployment rate throughout Europe (see, for example, Layard et al 1991 or Ljungqvist and Sargent 1995). There have also been claims that the current U.S. unemployment insurance program generates a large welfare loss for the economy (see, for example, Mortensen 1994).

In their book, Layard, Nickell and Jackman (1991) trace much of the recent European experience with unemployment to changes in UI programs in the European countries. They argue that the gradual increase in the “natural rate” of unemployment in several European countries can be explained by the increased generosity of their UI programs. In addition, they argue that much of the cross-country differences in unemployment can be attributed to differences in their UI programs. In fact, they estimate that approximately 91 percent of the variation across the major OECD industrial countries in the 1983-88 average unemployment rate can be explained merely by the variation in the generosity of labor market policies and the extent of collective bargaining coverage. Based on their results, Layard et al suggest a variety of reforms to combat Europe’s dual problems of high unemployment and long average duration of unemployment. For example, with respect to the U.K. they suggest reducing the potential duration of UI benefits, discarding policies that impose employment-adjustment costs on firms, and instituting subsidies to offset recruiting and training costs incurred by firms.

The purpose of the Layard et al book is to provide estimates of the impact of various labor market policies on unemployment and to suggest reforms. However, the authors make no attempt to link the employment effects that they estimate to economic welfare. Thus, it is hard to assess

whether European UI programs are welfare enhancing or debilitating. In addition, their analysis provides no guidance as to how the reforms they suggest would improve welfare when compared to the present programs.

In two recent papers, Mortensen (1994) and Millard and Mortensen (1995) improve on the Layard et al approach by estimating the welfare effects of a variety of labor market policies including unemployment insurance. They use a general equilibrium search model so as to capture the cost of UI through its impact on the aggregate unemployment rate. In addition to the tax burden it creates, UI generates economic costs for two main reasons. First, as we have already discussed, more generous UI lowers the opportunity cost of unemployment and leads to lower search effort by the jobless. This increases the equilibrium rate of unemployment and reduces output. Second, since more generous UI makes the unemployed less likely to accept new jobs, the wage that firms must offer rises, making production less profitable. This decreases the total number of jobs available in the economy. This job destruction effect further lowers employment, production, and welfare. As discussed above, this latter effect is absent from Baily's and Flemming's analyses since they do not model firm behavior.

For our purposes, the most important results from Mortensen (1994) and Millard and Mortensen (1995) concern the UI programs in the U.S. and the U.K. To estimate the impact of these programs, the authors calibrate their model using data on labor market flows in the U.S. during 1983-1992 and estimates of parameters that are obtained from the labor economics and macroeconomics literatures. Following Layard et al, they then recalibrate the model for the U.K. assuming that differences in the U.S. and U.K. unemployment experiences can be attributed to differences in their labor market policies and union coverage rates.

In both papers, welfare is measured by aggregate income net of search, recruiting, and training costs. With this measure, Mortensen (1994) estimates that a 50 percent reduction in the U.S. replacement rate would reduce the equilibrium rate of unemployment by 1.48 percentage points and increase net output by slightly less than one percentage point. He also estimates that a 50 percent reduction in the potential duration of benefits would decrease the equilibrium rate of unemployment by .78 percentage points while increasing welfare by about .5 percentage points.

As for the U.K., Millard and Mortensen estimate that the welfare cost imposed by its current UI program is roughly equal to 1.7 percent of net output, a large measure for dead weight loss. They also estimate that by limiting the benefit period to 2 quarters (as in the U.S.), the U.K. could increase welfare by more than one percentage point (and lower unemployment by over 2 percentage points). Moreover, if the employment-adjustment costs imposed by the government were also eliminated (as suggested by Layard et al), Mortensen and Millard estimate that welfare in the U.K. would rise by as much as 3.5 percent.

It is easy to infer from these results that the current UI programs in the U.S. and the U.K. impose significant welfare burdens on their economies. However, there is at least one serious drawback to these analyses. By using aggregate net income as their measure of welfare, the authors implicitly assume risk neutrality on the part of workers so that there is no need or desire for insurance of any kind. It follows that the positive aspects of UI—the fact that it provides desired insurance against employment risk—are given no weight in the welfare calculations. Thus, these two papers focus on the costs of the program while ignoring the benefits it provides. In the following two sections we offer a model that captures the costs of UI and also assumes risk aversion on the part of workers so that insurance is valuable. We then compare our results with those of Mortensen and

Millard and Mortensen to illustrate the importance of risk aversion in determining the generosity of an optimal UI program.<sup>3</sup>

### 3. A MODEL WITH RISK AVERSION AND JOB DESTRUCTION

In this section we provide a description of the model that we use to derive the optimal UI program. As we describe our model, we also point out the elements that are missing from each of the analyses described in Section 2. This should help clarify some of our criticisms of the earlier literature.

We follow the tradition in this literature by using a search model of the labor market. In order to focus on the benefits and costs of UI we model the behavior of a typical unemployed worker who is searching for a job and desires employment insurance. This worker earns a wage of  $w$  each period while employed and collects UI benefits of  $x$  per period while unemployed provided that she has not exhausted her benefits. Benefits are provided by the government to jobless workers who have been unemployed for no more than  $T$  periods. Thus, initially, we assume that all new job losers are eligible for UI. In section 4, we modify the model to account for the fact that the actual UI take-up rate is below 100 percent.

In our model, UI is funded by taxing all employed workers' earnings at a constant rate  $\tau$ . This assumption, common in the optimal UI literature, is used to capture the notion that the incidence of a UI tax is likely to be borne by workers.

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<sup>3</sup>Valdivia (1995) extends the Mortensen (1994) analysis to allow for risk aversion and finds optimal replacement rates of approximately 30 percent for the United States. However, this rate is derived assuming that benefits are offered indefinitely (as in the Baily/Flemming analyses). Fredriksson and Holmlund (1998) allow for risk aversion in a model in which the potential duration of benefits is finite and stochastic and the benefit rate is allowed to change once over the spell of unemployment. Their main goal is to argue that a two-tier system in which the benefit rate drops at some point in time dominates a program with a uniform replacement rate. With a uniform benefit structure, they find optimal replacement rates between 27 and 42 percent, depending on the degree of risk aversion.

We assume that unemployed workers choose search effort ( $p$ ) to maximize expected lifetime utility and that all workers are infinitely lived. As for firms, we assume that each firm hires at most one worker and that new firms enter the labor market until the expected profit from creating a vacancy is zero. Once a firm with a vacancy and an unemployed worker meet, they negotiate the wage. Following a well-established tradition in the search literature, we assume that the negotiated wage splits the surplus created by the job evenly (this will be made precise below). Total labor demand ( $F$ ) and search effort together determine equilibrium steady-state unemployment ( $U$ ).

The government's goal is to choose  $x$  and  $T$  to maximize aggregate expected lifetime utility. Increases in  $x$  and/or  $T$  provide unemployed workers with additional insurance but these increases also lower optimal search effort.<sup>4</sup> In addition, since a more generous UI program reduces the opportunity cost of unemployment, it increases the wage rate and makes it less profitable for a firm to create a vacancy. The reduction in search effort coupled with the destruction of job opportunities leads to an increase in unemployment. The optimal government policy must balance these costs and benefits.

In terms of the literature reviewed above, our approach is similar to that of Mortensen (1994) and Millard and Mortensen (1995), except that we assume risk aversion on the part of workers. Our work could also be viewed as an extension of Baily (1978) and Flemming (1978) in which we (a) make the potential duration of benefits variable, (b) take into account the fact that the UI take-up rate is below 100 percent, and (c) model labor demand so that the job destruction effects of UI are taken into account.

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<sup>4</sup>A more generous UI program may also induce entry into the work force. We return to this issue in section 4.D when we extend the model to allow for worker heterogeneity.

We describe the model in three steps. First, we show how to determine expected lifetime utility for all agents in the economy and use these measures to define welfare. We also show how these measures may be used to determine optimal search effort for unemployed workers. Second, we show how total labor demand and search effort can be combined to determine unemployment. Finally, we introduce our model of firm behavior and show how total labor demand and the wage are determined.

A few words about our notation are in order. Throughout the analysis we define variables such as search effort, expected lifetime utility, reemployment probabilities, et cetera that depend upon the employment status of the worker. In each case, we use sub-scripts on the variables to denote the employment status with  $w$  representing employed workers,  $t$  denoting unemployed workers in their  $t^{\text{th}}$  period of search, and  $x$  denoting unemployed workers who have exhausted their benefits. For example, using  $m$  to denote the reemployment probability,  $m_t$  represents the reemployment probability for an unemployed workers in the  $t^{\text{th}}$  period of search, while  $m_x$  represents the reemployment probability for an unemployed worker who has exhausted her benefits.

#### A. Expected Lifetime Utility, Search Effort, and Welfare

We use  $V_j$  to denote expected lifetime utility for a worker in employment state  $j$  ( $j = w$  if employed,  $t$  if unemployed for  $t$  periods, and  $x$  if unemployed and benefits have been exhausted). In addition, we use  $u(\cdot)$  to represent the agents' common utility function. We assume that per period utility takes the form  $u(C) - c(p)$  with  $C$  denoting consumption,  $c(p)$  denoting the cost of search, and  $p$  denoting search effort (if unemployed). We assume that  $c(p)$  is a convex function and that  $c(0) =$

0. We begin by assuming that agents cannot save so that in any given period consumption equals income. In Section 5 we discuss how relaxing this assumption affects our results.

For employed workers, current income consists of two components—labor income,  $w(1 - J)$ , which is equal to the wage net of taxes, and non-labor income,  $z_w$ , which is equal to workers' share of the aggregate profits earned by the firms. Thus, current utility is given by  $u[w(1 - J) + z_w]$ . Obviously, employed agents incur no search costs. To determine expected lifetime utility, we must also consider the worker's future prospects. Let  $s$  denote the probability that in any given period the worker will lose her job. Then, with probability  $(1 - s)$ , the worker's expected future lifetime utility will continue to be  $V_w$  (since she remains employed). With probability  $s$ , the worker loses her job and her expected future lifetime utility falls to  $V_1$ . It follows that,

$$(1) \quad V_w = u[w(1-J)+z_w] + [sV_1+(1-s)V_w]/(1+r).$$

Note that future utility is discounted at rate  $(1+r)$  with  $r$  denoting the interest rate.

Turn next to the unemployed. For them, current income is equal to the sum of unemployment insurance (if benefits have not yet been exhausted) and profits. We use  $z_u$  to denote a typical unemployed worker's share of aggregate profits. Future income depends on future employment status. We use  $m$  to denote reemployment probabilities so that with probability  $m_t$  the worker finds a job and can expect to earn  $V_w$  in the future, while with probability  $(1 - m_t)$  she remains unemployed and can expect to earn  $V_{t+1}$  in the future. Thus,

$$(2) \quad V_t = u[x+z_u] - c(p_t) + [m_t V_w + (1-m_t)V_{t+1}]/(1+r) \quad \text{for } t = 1, \dots, T.$$

$$(3) \quad V_x = u[z_u] - c(p_x) + [m_x V_w + (1-m_x)V_x]/(1+r).$$

We can now define welfare ( $W$ ). Let  $U_t$  represent the number of workers who have been unemployed for  $t$  periods and define  $U_x$  analogously for UI-exhaustees. Then, if we define  $J$  to be the total number of jobs held in the steady-state equilibrium and aggregate expected lifetime utility across all agents, we obtain

$$(4) \quad W = J V_w + U_x V_x + \sum_t U_t V_t.$$

Finally, since search effort is chosen to maximize expected lifetime income we have,

$$(5) \quad p_t = \arg \max V_t \quad \text{for } t = 1, \dots, T.$$

$$(6) \quad p_x = \arg \max V_x.$$

It will become clear in sub-section B below that the reemployment probability ( $m$ ) is an increasing function of search effort ( $p$ ).

## B. Determining Unemployment

In this sub-section we show how total labor demand ( $F$ ) and search effort ( $p$ ) can be combined to determine equilibrium unemployment. To do so, we first show how to determine steady-state unemployment once the reemployment probabilities have been determined. Second, we show how the reemployment probabilities vary with search effort, labor demand, and other features of the labor market.

Formally, we use  $L$  to denote total labor supply. Then, since every worker is either employed or unemployed, we have



$$(7) \quad L = J + U.$$

In addition, given our definitions of  $U_t$  and  $U_x$  we can write total unemployment as

$$(8) \quad U = \sum_t U_t + U_x.$$

Turn next to the firms. For simplicity, we assume that each firm provides only one job opportunity.<sup>5</sup> Thus,  $F$  denotes both the total number of firms and the total number of jobs available at any time. Each job is either filled or vacant. If we let  $V$  denote the number of vacancies in a steady-state equilibrium, it follows that

$$(9) \quad F = J + V.$$

We can now describe the dynamics of the labor market and the conditions that must hold if we are in a steady-state equilibrium. These conditions guarantee that the unemployment rate and the composition of unemployment both remain constant over time. Recall that  $s$  is defined to be the economy's separation rate—that is,  $s$  denotes the probability that an employment relationship will dissolve in any given period. In addition, recall that reemployment probabilities are denoted by the  $m$  terms. Then, for any given worker, there are  $T + 2$  possible employment states— $U_1, U_2, \dots, U_T, U_x$ , and  $J$ . If employed (i.e., if in state  $J$ ) the worker faces a probability  $s$  of losing her job and moving into state  $U_1$ . If unemployed for  $t$  periods (i.e., if in state  $U_t$ ), the worker faces a probability of  $m_t$  of finding a job and moving into state  $J$ . With probability  $(1 - m_t)$  this worker remains unemployed and

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<sup>5</sup>This assumption is commonly used in general equilibrium search models (see, for example, Diamond 1982 or Pissarides 1990). Alternatively, we could simply assume that each firm recruits for and fills each of its many vacancies separately.

moves on to state  $U_{t+1}$ . Finally, UI-eligible exhaustees face a reemployment probability of  $m_x$ , in which case they move into state J. Otherwise, they remain in state  $U_x$ .

In a steady-state equilibrium the flows into and out of each state must be equal so that the unemployment rate and its composition do not change over time. Using the above notation, the flows into and out of state  $U_1$  are equal if

$$(10) \quad sJ = U_1.$$

The flows into and out of state  $U_t$  (for  $t = 2, \dots, T$ ) are equal if

$$(11) \quad (1-m_{t-1})U_{t-1} = U_t$$

Finally, the flows into and out of state  $U_x$  are equal if

$$(12) \quad (1-m_T)U_T = m_x U_x.$$

In each case, the flow into the state is given on the left-hand-side of the expression while the flow out of the state is given on the right-hand-side.

Equations (7)-(12) define the dynamics of the labor market given the reemployment probabilities and total labor demand. We must now explain how search effort translates into a reemployment probability for each unemployed worker. As described above, each unemployed worker chooses search effort ( $p$ ) to maximize expected lifetime utility. Search effort is best thought of as the number of firms a worker chooses to contact in each period of job search. For workers who contact fewer than one firm on average,  $p_t$  could be thought of as the probability of contacting any firm. Once a worker contacts a firm, she files a job application if the firm has a vacancy. Since there

are  $F$  firms and  $V$  of them have vacancies, the probability of contacting a firm with a vacancy is  $V/F$ . Finally, once all applications have been filed, each firm with a vacancy fills it by choosing randomly from its pool of applicants. Thus, if  $N$  other workers apply to the firm, the probability of a given worker getting the job is  $1/(N+1)$ . Since each other worker either does or does not apply,  $N$  is a random variable with a Poisson distribution with parameter  $\lambda$  equal to the average number of applications filed at each firm. It is straightforward to show that this implies that the probability of getting a job offer conditional on having applied at a firm with a vacancy is  $(1/\lambda)[1 - e^{-\lambda}]$ . The reemployment probability for any given worker is then the product of these three terms—the number of firms contacted, the probability that a given firm will have a vacancy, and the probability of getting the job conditional on having applied at a firm with a vacancy:

$$(13) \quad m_t = p_t(V/F)(1/\lambda)[1 - e^{-\lambda}] \quad \text{for } t = 1, \dots, T$$

$$(14) \quad m_x = p_x(V/F)(1/\lambda)[1 - e^{-\lambda}]$$

where

$$(15) \quad \lambda = \{\sum_t p_t U_t + p_x U_x\}/F.$$

These equations define the reemployment probabilities of workers as a function of search effort and the length of time that they have been unemployed. Note that for any given worker, the search effort of other workers affects that worker's reemployment probability through  $\lambda$ .

Given the levels of search effort and expected lifetime utilities defined by (1)-(6), equations (7)-(15) can be solved for equilibrium unemployment ( $U$ ), its composition ( $U_t$  for  $t = 1, \dots, T$  and  $U_x$ ),

and the reemployment probabilities ( $m_t$  for  $t = 1, \dots, T$  and  $m_x$ ). If we were to stop developing the model at this point, treating  $F$  and  $w$  as exogenous, we would have a model almost identical to the one used by Flemming (1978). In fact, there would be only two real substantive differences between the models—Flemming allows workers to save while employed while we do not, and Flemming assumes that UI is offered indefinitely while we assume that it is only offered for  $T$  periods. Below we make the number of firms ( $F$ ) and the wage ( $w$ ) endogenous and add UI-ineligible workers to the model.

### C. Firms

To make the number of firms endogenous we assume that firms enter the market until the expected profit from doing so equals zero. When a firm enters the market, it creates a vacancy and starts to accept applications from unemployed workers to fill it. Once the vacancy is filled, the firm produces and sells output as long as its vacancy remains filled. If the firm loses its worker, it must restart the process of filling its vacancy.

We use  $\Pi_v$  to denote the expected lifetime profit for a firm that currently has a vacancy and use  $\Pi_j$  to represent the expected lifetime profit for a firm that has filled its vacancy. Thus, when a firm enters the market and creates a vacancy it can expect to earn  $\Pi_v$  in the future. Once it fills its vacancy, its expectations about future profits rise to  $\Pi_j$ . Firms enter until

$$(16) \quad \Pi_v = 0.$$

To calculate  $\Pi_v$  and  $\Pi_j$  we follow the same procedure that was used to determine expected lifetime utilities—we consider the current and future prospects of typical firms. Let  $q$  denote the

probability of filling a vacancy, use  $K$  to represent the cost of maintaining a vacancy, and let  $R$  denote the net revenue earned by a producing firm (net of  $K$ ). Then, current profit for a firm with a vacancy is  $-K$  while current profit for a producing firm is  $R - w$ . Now consider their future prospects. A firm that has an opening fills it with probability  $q$ , in which case its expected lifetime profits rise to  $\Pi_j$ . With the remaining probability the vacancy remains open and the firm continues to expect to earn  $\Pi_v$ . Thus,

$$(17) \quad \Pi_v = -K + [q\Pi_j + (1-q)\Pi_v]/(1+r).$$

A firm that has already hired a worker keeps that worker with probability  $(1-s)$  and continues to earn  $\Pi_j$ . With probability  $s$ , it loses its worker and sees its expected profits fall to  $\Pi_v$ . Thus,

$$(18) \quad \Pi_j = R - w + [s\Pi_v + (1-s)\Pi_j]/(1+r).$$

Note that, as before, future profits are discounted at rate  $(1+r)$ .

The probability of filling a vacancy,  $q$ , depends on the number of firms competing for the unemployed ( $V$ ), the number of unemployed workers ( $U$ ) and the search effort of workers. In any given period the number of unemployed workers who find new jobs is equal to  $\sum_t m_t U_t + m_x U_x$  while the number of vacancies that are filled is equal to  $qV$ . Since these values must be equal, we have

$$(19) \quad q = [\sum_t m_t U_t + m_x U_x]/V.$$

Note that the search effort of workers enters (19) through the reemployment probabilities.

The next step is to use  $\Pi_v$  and  $\Pi_j$  to determine the profits that are distributed to workers in each period in the form of dividends ( $\theta_w$  for the employed and  $\theta_u$  for the unemployed). Since there

are  $J$  jobs filled in equilibrium with each one generating  $\Pi_j$  in expected lifetime profits, aggregate expected lifetime profits are  $J\Pi_j$ . Thus, the aggregate per period profits are equal to  $rJ\Pi_j/(1+r)$ . These profits must be distributed to workers each period. We assume that these profits are distributed evenly to employed workers with the unemployed receiving nothing. It follows that  $\theta_w = rJ\Pi_j/(1+r)J = r\Pi_j/(1+r)$  and  $\theta_u = 0$ . We make this assumption for the following reason. Suppose that the government were to reduce the generosity of the UI program, and that aggregate profits increased as a result. If the unemployed were to receive a share of these profits, this increase in non-labor income could swamp the decrease in UI and leave the unemployed better-off. Since most unemployed receive little income from such non-labor sources, we assume that all profits go to the employed. In addition, with this assumption, the optimal UI program in our model will be less generous than one derived in a model in which the unemployed receive a share of firms' profits. Since this assumption biases downward our estimates of the optimal replacement rate and potential duration of benefits, and since we conclude that current programs are probably not generous enough, this assumption seems innocuous.

The final step in developing our model is to explain how the wage is determined. Following the general equilibrium search literature (see, for example, Diamond 1982 or Pissarides 1990), we assume that the firms and workers split the surplus created by the representative job evenly. When firms fill a vacancy their expected profits rise from  $\Pi_v$  to  $\Pi_j$ . When an average worker becomes reemployed his expected lifetime utility rises from  $V_u$  to  $V_w$ , where  $V_u$  denotes the average expected lifetime utility for an unemployed worker.<sup>6</sup> That is,

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<sup>6</sup>Strict application of the Nash bargaining solution would require us to use a different threat point for unemployed workers who have been unemployed a different length of time. For example, the threat point for an unemployed worker in her  $t^{\text{th}}$  period of search would be  $V_{t+1}$  while the threat point for an unemployed worker who has exhausted her benefits would be  $V_x$ . This would result in the firm paying different wages to workers with different unemployment histories and would imply that the firm would prefer to hire a long-term unemployed worker rather than some one who had been

$$(20) \quad V_u = [\sum_t U_t V_t + U_x V_x]/U.$$

It follows that the total surplus created by the average job when measured in dollars is  $(\Pi_J - \Pi_V) + (V_w - V_u)/MU_I$  where  $MU_I$  represents the worker's marginal utility of income and allows us to transform the workers gain,  $V_w - V_u$ , which is measured in utility, into an appropriate dollar value. This surplus is split evenly between the firm and its employee if the wage satisfies

$$(21) \quad \Pi_J - \Pi_V = (V_w - V_u)/MU_I.$$

In summary, when we model firms the number of firms demanding labor ( $F$ ) is determined by (16) while the equilibrium wage is determined by (21).

The government's problem is to choose  $x$  (the UI benefit level) and  $T$  (the potential duration of benefits) to maximize welfare ( $W$ , as given in eq. 4) subject to the constraint that its budget balances. Since there are  $J$  employed workers each earning a wage of  $w$ , total tax revenue is equal to  $Jw\tau$ . In equilibrium,  $U - U_x$  unemployed workers receive benefits of  $x$  each period. Thus, the total cost of the program is  $(U - U_x)x$ . For the budget to balance it must be the case that

$$(22) \quad (U - U_x)x = Jw\tau.$$

As noted above an increase in  $x$  or  $T$  increases the level of insurance provided to unemployed workers, but both increase unemployment and require that  $\tau$  increase in order to fund the expanded program.

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unemployed for a relatively short time (since the firm could offer a lower wage to the worker who has been unemployed for a long time). This would greatly complicate the analysis without adding any insight into the design of an optimal UI program. Thus, for simplicity, we assume that the threat point is the same for all unemployed workers and that this threat point is the average utility of all unemployed agents. This allows us to capture the notion that the average wage will rise when the utility of the unemployed increases.

This completes the description of our model. In structure it is similar to that of Mortensen (1994) and Millard and Mortensen (1995). The major difference is in the measurement of welfare—whereas they use aggregate income net of search, recruiting, and training costs as their measure of welfare, we use aggregate expected lifetime utility. These two measures are identical if agents are risk neutral. However, if the utility function is concave, so that agents are risk averse, the measures differ. As we argued above, we feel that it is important to assume risk aversion since to do otherwise implies that unemployment insurance has no value to workers.

#### D. Properties of Equilibrium

Before we turn to optimal policy, it is useful to describe the structure of equilibrium and some of its comparative dynamic properties. It is straightforward to show that in a steady-state equilibrium  $V_w > V_1 > V_2 > \dots > V_T > V_x$ . That is, expected lifetime income is highest for employed workers, lowest for unemployed workers who have exhausted their benefits, and decreasing in the number of weeks that a worker has been unemployed. Intuitively, workers in the early stages of a spell of unemployment have more weeks to find a job before they have to worry about exhausting their UI benefits. Because of this, workers who have recently become unemployed will not search as hard as those who have been unemployed for a longer period of time—that is, optimal search effort will be increasing in the number of weeks of unsuccessful search ( $p_1 < p_2 < \dots < p_T < p_x$ ).

A decrease in UI benefits ( $x$ ) or the potential duration of benefits ( $T$ ) decreases the level of insurance offered unemployed workers and triggers an increase in search effort by all UI-eligible workers (and therefore lowers unemployment). Either change results in a decrease in  $V_t$  for all  $t$ , but decreases in  $x$  and  $T$  have opposite effects on the probability of exhausting benefits. A decrease in



$x$  makes it less likely that a worker will exhaust her UI benefits before finding a job (since she searches harder). But a decrease in  $T$  makes it more likely that benefits will be exhausted since the time horizon over which benefits are offered has been shortened (this is true in spite of the fact that search effort increases as  $T$  falls). Of course, increases in  $x$  or  $T$  lead to the opposite effects.

Changes in the UI program also have implications for firm behavior and labor demand. Since increases in either  $x$  or  $T$  reduce the cost of being unemployed, they make workers less willing to search for and/or accept jobs. This results in an increase in  $V_u$  and forces firms to increase the wage that they offer their new employees. This increased wage makes production less profitable and results in fewer firms and job opportunities. This job destruction effect increases unemployment and lowers net output.

#### E. Calibration

In order to determine the optimal UI program we must choose values for the parameters of the model, solve for the equilibrium generated by each pair of policy parameters ( $x$  and  $T$ ), and compare the levels of welfare achieved in the different equilibria. Assuming that we choose realistic values for the parameters, this exercise should give us some idea as to the ranges in which the optimal level of benefits and the optimal potential duration of benefits lie.

The parameters of the model are the separation rate ( $s$ ), the interest rate ( $r$ ), the size of the labor force ( $L$ ), the search cost function ( $c(p)$ ), the revenue earned by producing firms ( $R$ ), the cost of maintaining a vacancy ( $K$ ), and the utility function,  $u(C)$ . Since we are interested in varying the degree of risk aversion, we calibrate the model separately for a variety of different utility functions and compare the optimal programs that result.

We calibrate the model in two steps. First, we treat the model introduced in sub-sections A and B as if it were self-contained—that is, we treat the number of firms ( $F$ ) and the wage ( $w$ ) as if they were parameters of the model. To calibrate this portion of the model we rely on data collected to analyze the Illinois Reemployment Bonus Experiment. Since we have discussed calibration of this abbreviated model in detail elsewhere (see, for example, Davidson and Woodbury 1991, 1993, 1997), we provide only a short description here. Briefly, the abbreviated model is calibrated so that its predictions concerning the impact of a reemployment bonus offered to unemployed workers matches what was observed in the experiment for workers who were eligible for regular state benefits in Illinois (Davidson and Woodbury, 1991). By treating  $F$  and  $w$  as fixed, we are implicitly assuming that the Illinois experiment had no wage or job creation/job destruction effects. In fact, the data indicate that there were no wage effects from the reemployment bonus (Woodbury and Speigelman 1987). Given that the bonus experiment was temporary and limited in scope, it seems reasonable to assume that there were no significant changes in the number of firms seeking workers as a result of the bonus.

In the second step, we expand the model (by adding sub-section C) so that  $F$  and  $w$  become endogenous. This adds two new parameters to the model— $R$  (the revenue earned by the firm when producing) and  $K$  (the cost of maintaining a vacancy). These values are chosen so that the full model yields (a) a value for  $w$  that matches the data collected in Illinois, and (b) values for  $F$  that lie in the range predicted by the abbreviated model in the first stage of calibration.

When considering the abbreviated model (sub-sections A and B), the parameters of interest are the separation rate ( $s$ ), the interest rate ( $r$ ), the wage ( $w$ ), the number of firms ( $F$ ), the size of the labor force ( $L$ ), and the search cost function ( $c(p)$ ). We can obtain an estimate for  $s$  from the existing

literature on labor market dynamics. Ehrenberg (1980) and Murphy and Topel (1987) provide estimates of the number of jobs that break-up in each period. If we measure time in 2-week intervals, their work suggests that  $s$  lies in the range of .007 to .013. For the interest rate we set  $r = .008$  which translates into an annual discount rate of approximately 20 percent. Since our previous work (Davidson and Woodbury 1991, 1993) suggests that results from this model are not sensitive to changes in  $r$  over a fairly wide range, this is the only value for the interest rate that we consider.<sup>7</sup>

For  $F$  and  $L$  we begin by noting that our model is homogeneous of degree zero in  $F$  and  $L$  so that we may set  $L = 100$  without loss of generality. If we then vary  $F$  holding all other parameters fixed we can solve for the equilibrium unemployment and vacancy rates. Abraham's (1983) work suggests that the ratio of unemployment to vacancies ( $U/V$ ) varies between 1.5 and 3 over the business cycle. Although the actual values of  $U$  and  $V$  depend on the other parameters, we find that to obtain such values for  $U/V$  in our model with  $L = 100$ ,  $F$  must lie in range of 95 to 97.5. Thus, in the second stage of the calibration, we must choose values for  $R$  and  $K$  such that  $F$  lies in the range 95-97.5.

The remaining parameters in sub-sections A and B are the wage rate and the search cost function. For these values we turn to the data and results from the Illinois Reemployment Bonus Experiment. In the Illinois experiment a randomly selected group of new claimants for UI were offered a \$500 bonus for accepting a new job within 11 weeks of filing their initial claim. The average duration of unemployment for these bonus-offered workers was approximately .7 weeks less than the average unemployment duration of the randomly selected control group (Davidson and Woodbury 1991). In our previous work, we estimated the parameters of the search cost function that

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<sup>7</sup>See footnotes 13 and 14 of Davidson and Woodbury (1997) for more details on the sensitivity of our results with respect to the interest rate.

would be consistent with such behavioral results. That is, we assumed a specific functional form for  $c(p)$  and then solved for the parameters that would make the model's predictions match the outcome observed in the Illinois experiment. The functional form that we used was  $c(p) = cp^z$ , where  $z$  denotes the elasticity of search costs with respect to search effort. The values for  $c$  and  $z$  that make the model's predictions exactly match what occurred in Illinois depend upon the utility function that is assumed. For example, if we assume that the utility function is linear in consumption, then our results indicated that for the average bi-weekly wage rate observed in Illinois (\$511), the values of  $c$  and  $z$  that are consistent with the Illinois experimental results are  $c = 338$  and  $z = 1.23$ . On the other hand, if the utility function takes the form  $u(C) = \ln(C)$ , we find that the values of  $c$  and  $z$  that are consistent with the Illinois experimental results are  $c = 2.05$  and  $z = 1.38$ .

Now consider the second stage of calibration. In order to make  $F$  and  $w$  endogenous, we add the equations in sub-section C to the model. This adds two new parameters,  $R$  and  $K$ . From the Illinois data we know that the average bi-weekly wage is \$511, and from stage one of the calibration we know that  $F$  must lie in the range 95 to 97.5. Thus, we set  $x$  and  $T$  equal to their Illinois values—\$242 for the average bi-weekly UI benefit ( $x$ ), and 14 for the potential duration of benefits ( $T$ , since each period equals 2 weeks)—and then solve the model to determine what values of  $R$  and  $K$  would lead the model to predict that  $w = \$511$  and that  $F$  would fall in the range 95-97.5. Of course, the values of  $R$  and  $K$  depend upon the assumed form of the utility function. If the utility function is linear in consumption, then when  $R = 724$  and  $K = 2417$  the model predicts that  $w = 511$  and  $F = 96.25$ . On the other hand, if  $u(C) = \ln(c)$ , then when  $R = 1469$  and  $K = 10863$  the model predicts that  $w = \$511$  and  $F = 96.25$ .

Once the calibration is complete, we set the parameters at the calibrated levels and solve for the welfare maximizing values of  $x$  and  $T$ . Once we have solved for the optimal values for  $x$  and  $T$  in one case, we vary the parameters over the ranges described above to test for the sensitivity of our results with respect to each parameter. As we show below, our results are largely insensitive to changes in all of the parameters except the degree of risk aversion. Thus, for most of our results we focus on our “reference case” in which  $s = .010$  and  $R$  and  $K$  are chosen so that  $F = 96.25$  and  $U/V = 2$ .

#### 4. RESULTS

In this section we begin by reviewing results from our earlier work (Davidson and Woodbury 1997), in which we solved for the optimal UI program in the abbreviated model outlined in sections 3.A and 3.B. These results are best thought of extensions of Baily’s and Flemming’s work to an environment in which (a) the potential duration of benefits can vary and be controlled by the government, and (b) not all unemployed workers are eligible for UI. Next, we present new results concerning optimal UI when firm behavior is explicitly added to the model as in section 3.C. This allows us to examine how our initial results must be modified when the job destruction effects of more generous UI programs are taken into account. Finally, we extend the model once more in order to allow for worker heterogeneity and show how including workers with different labor market experiences in the model alters the results.

## A. Optimal Potential Duration of Benefits without Job Destruction

The most surprising result from our earlier analysis is that in the abbreviated model the optimal potential duration of benefits is infinite—that is, the government should offer benefits indefinitely to all unemployed UI-eligible workers. Although some details are omitted from the following reasoning,<sup>8</sup> the crux of the argument is as follows. Agents facing employment risk would prefer a program that allows them to smooth consumption as much as possible across spells of unemployment. Thus, given a choice between two UI programs that provide the same level of total benefits, agents would choose the program that does the best job of smoothing consumption. With this in mind, consider the following two programs—the first program offers a benefit of  $x$  for  $T$  periods while the second program offers a benefit of  $x'$  for  $T+1$  periods where  $x' < x$  and is chosen so that the two programs provide the same level of total unemployment benefits. Thus, the first program offers higher benefits per period but for fewer periods. The key to the argument is to note that the second program allows for greater consumption smoothing—in moving from the first program to the second program, benefits are lowered during the least adverse states of unemployment (i.e., the initial phase) and increased in one of the most adverse states (period  $T+1$  in which no benefits are offered in the first program) with total benefits provided remaining the same. In other words, by accepting slightly decreased benefits (and consumption) during the first  $T$  periods of unemployment, the unemployed can insure that benefits will not completely disappear for an additional period. Thus, all risk-averse unemployed workers prefer the second program.<sup>9</sup> Since this reasoning holds for all finite  $T$ , it follows that in an optimal UI program  $T$  must equal infinity.

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<sup>8</sup>See Davidson and Woodbury (1997) for details.

<sup>9</sup>In the terminology of decision making under uncertainty, the unemployed prefer the second program to the first program because it makes unemployment “less risky” in the Rothschild-stiglitz sense.

This result has important implications for some of the work reviewed in section 2. Most importantly, it implies that the conclusions reached by Baily and Flemming are misleading. Since both authors use models in which it is assumed that benefits are offered indefinitely and since, in their models, it is indeed optimal to provide benefits indefinitely, the optimal replacement rates that they derive are correct—without savings, the optimal replacement rate is in the 60-70 percent range, and, with savings but imperfect capital markets, the optimal replacement rate is in the 40-50 percent range.<sup>10</sup> However, these rates are optimal only if they are offered indefinitely. Thus, the conclusion that Baily and Flemming reach—that the 50 percent replacement rate in most states of the U.S. is probably too high—cannot be right, since the U.S. offers this rate for only 26 weeks. In fact, when we solve for the optimal replacement rate in our abbreviated model with T set exogenously at 26 weeks, we find that the optimal replacement rate is above 1. It follows that if one ignores the job destruction effect of UI, the current U.S. unemployment insurance program is not generous enough.

It is important, however, not to place too much emphasis on this result. That is, we must remember the setting in which it was derived—a model in which the job destruction effects of UI were ignored. In fact, as we show below, when the job destruction effects are taken into account, this result no longer holds. For this reason, we do not believe that an optimal UI program would indeed be characterized by an unlimited potential duration of benefits. However, what this result does indicate is that an optimal UI program is more likely to be characterized by low benefits and a long potential duration of benefits than a program with high benefits and a short potential duration of benefits (as in the U.S.). The intuition behind this result is clear—programs with long potential

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<sup>10</sup>It is important to note that our abbreviated model yields almost identical predictions concerning optimal replacement rates, as shown in Table 5 of Davidson and Woodbury (1997).

durations of benefits lead to smoother consumption paths and therefore reduce the risk associated with unemployment more than programs with shorter potential durations.

## B. Optimal Replacement Rates with UI-Ineligibles in the Model

Our second extension of the Baily and Flemming analyses was to take account of the fact that not all unemployed workers are eligible to collect UI. For example, for the U.S., Blank and Card (1991) report that over 50 percent of the unemployed are ineligible for UI and that of those who are eligible, only 75 percent claim their benefits.<sup>11</sup> Storer and Van Audenrode (1995) estimate that 85-90 percent of UI eligibles in Canada actually claim their benefits. Layard et al (1991) report that in the U.K. up to 30 percent of the unemployed are not eligible to collect UI benefits. These facts have important implications for the optimal replacement rate since more generous UI has positive spill-over effects on UI-ineligibles. The reasoning is as follows. If the government institutes a more generous UI program, UI-eligibles respond by searching less hard for employment. Assuming that UI-eligibles and UI-ineligibles compete for some of the same jobs, this reduces the competition that UI-ineligibles face for those jobs and increases their reemployment probabilities. The existence of these positive spill-over effects implies that ignoring the ineligibility of many workers to collect UI will underestimate the optimal replacement rate. Thus, Baily and Flemming's estimates of the optimal replacement rate are biased downwards.

To determine the optimal replacement rate when these positive spill-over effects are present, we extended the abbreviated model of sections 3.A and 3.B to allow for UI-ineligibility. Briefly, UI-ineligibles were modeled in exactly the same manner as other workers except that they were not

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<sup>11</sup>McCall (1995) reports that only 60 percent of UI eligibles claimed their benefits between 1984 and 1990.



allowed to collect UI while unemployed. For example, equations analogous to (2) and (3) were used to define the expected lifetime utility for an unemployed UI-ineligible worker, and an equation analogous to (1) was used to define the expected lifetime utility for an employed UI-ineligible worker. To be precise, let  $V_i$  represent the expected lifetime utility for an unemployed UI-ineligible worker, let  $V_{wi}$  denote the expected lifetime utility for an employed UI-ineligible worker, and use the sub-script  $i$  on all other variables to denote UI-ineligibility. Then, the same logic used to derive (1)-(3) gives:

$$(23) \quad V_{wi} = u[w(1-\tau)+\theta_w] + [sV_i+(1-s)V_{wi}]/(1+r)$$

$$(24) \quad V_i = u[\theta_u] - c(p_i) + [m_iV_{wi}+(1-m_i)V_i]/(1+r).$$

Optimal search effort for UI-ineligibles is then the value of  $p_i$  that maximizes  $V_i$ :

$$(25) \quad p_i = \arg \max V_i.$$

The remaining equations of the model can be modified in a similar fashion (see Davidson and Woodbury 1997 for details) with only one new parameter added - the proportion of the unemployed who are ineligible for UI. Following Blank and Card (1991) we set this value equal to .6 for our reference case, and then vary it throughout the analysis from 0 (all unemployed workers eligible) to .6 to check the sensitivity of our results.

We find, as expected, that including UI-ineligibles in the model does increase the optimal replacement rate. Depending upon the values of the other parameters, we find that the spill-over effects of UI on UI-ineligibles increases the optimal replacement rate by 6 to 10 percentage points. Thus, if agents cannot save and the job destruction effects of UI are ignored, an optimal UI program

offers a replacement rate in the 65-75 percent range indefinitely. If, on the other hand, agents can save (but the job destruction effects of UI are still ignored), then an optimal UI program offers a replacement rate in the 45-55 percent range indefinitely.

This completes the description of our earlier results, all of which were derived assuming that utility is linear in consumption. If we had also assumed that search costs were linear in effort, this would have been equivalent to assuming risk neutrality, and there would have been no demand for UI. However, since we assumed that search costs were convex in effort, each individual's optimization problem was concave in the choice variable and thus, each agent was risk averse.

To see how increasing the degree of risk aversion affects these results, we now recalibrate the model for two different utility functions, namely  $u[C] = \ln(C)$  and  $u[C] = \sqrt{C}$ , and rederive the optimal replacement rate in each case. The log utility function is characterized by constant Arrow-Pratt relative risk aversion equal to one and is chosen since it is identical that used by Baily (1978). The square root utility function is characterized by constant Arrow-Pratt relative risk aversion equal to one-half and is used since its measure of risk aversion falls mid-way between the linear and log utility functions. Surprisingly, in the model without job destruction, we find that the degree of risk aversion does not make much difference—optimal replacement rates fall, but only by a relatively small amount: about 6 percentage points when we go from the linear to the log utility function and about 2 percentage points when we go from the linear to the square root utility function.<sup>12</sup> The reason is that in recalibrating the model with the new utility functions, the values of the parameters change so that

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<sup>12</sup>Although the optimal replacement rate falls slightly when risk aversion is increased, the true level of insurance provided by these benefits is higher than the level provided under linear utility. The level of insurance can be measured by comparing utility while receiving UI benefits with utility while employed. With a linear utility function this ratio is identical to the replacement rate. But with concave utility, this ratio and the replacement rate are quite different. For example, with log utility a replacement rate of .60 (implying a bi-weekly benefit of \$305 for a worker with pre-layoff bi-weekly wages of \$511) gives a utility ratio of .91. Thus, optimal insurance is increasing in the degree of risk aversion.

the model once again yields predictions that are consistent with the Illinois data.<sup>13</sup> For example, as we make the agents in the model more risk averse, the degree of convexity of the search cost function must also increase so that the model still yields correct predictions about a reemployment bonus. Since we recalibrate the model for each utility function so that the reemployment bonus impact is the same, it is not surprising that the models suggest similar optimal UI programs.<sup>14</sup>

In summary, our earlier work focused on extending the analyses offered by Baily and Flemming in two ways. They assumed that UI benefits would be offered indefinitely and that all agents are eligible for UI. We showed that both of these assumptions biased their results in favor of less generous UI programs and led them to draw misleading conclusions. In the next sub-section we discuss how further extending the model to allow for the job destruction effects of UI forces us to further modify our conclusions concerning an optimal UI program. This is accomplished by endogenizing firm behavior.

### C. Job Destruction and Risk Aversion

When firm behavior is endogenized, UI has several effects that have not been considered to this point. First, if a more generous UI program is offered, the average expected lifetime income for the unemployed ( $V_u$ ) rises and this triggers an increase in the equilibrium wage. This higher wage lowers profits for producing firms ( $\Pi_j$ ) and lowers the expected lifetime profit for a firm creating a vacancy ( $\Pi_v$ ). This results in fewer firms ( $F$ ) and fewer job opportunities. In terms of welfare, per

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<sup>13</sup>When Baily and Flemming changed the degree of risk aversion in their models they held all other parameters fixed.

<sup>14</sup>When we calibrate the model for the square root utility function in the reference case ( $s = .010$ ) we obtain the following values for the key parameters— $c = 11.2$ ,  $z = 1.284$ ,  $R = 881$ ,  $K = 4196$ .

period income for the employed could rise or fall (since the wage is increasing while non-labor income from firms is falling) while unemployment unambiguously rises due to the job destruction effect. Thus, in a model with endogenous labor demand the optimal UI program is likely to be less generous than the optimal UI program in a model in which firm behavior is ignored, and the size of the job destruction effect determines just how much less generous it will be.

Our results indicate that, in almost all cases, the job destruction effect is large enough to overturn the result that it is optimal to offer UI benefits indefinitely. To see why, return to the earlier argument concerning the potential duration of benefits. We argued that for any UI program in which  $T$  were finite there would exist another UI program with longer potential duration of benefits and lower benefits that would cost the same to finance and would be strictly preferred by all agents. Thus, it would always be possible to increase  $T$  and raise welfare. This argument no longer holds when labor demand is endogenous since increasing  $T$  in this manner reduces the number of job opportunities and increases unemployment. This negative effect (the decrease in job opportunities) must be weighed against the positive impact of smoothing consumption to determine if the increase in  $T$  raises welfare. We find that for almost all levels of risk aversion, the job destruction effect of increasing  $T$  eventually outweighs the consumption smoothing effect of increasing  $T$  so that benefits should eventually be cut off.

The point at which the government should stop providing benefits depends heavily on the degree of risk aversion. We consider three cases.

Case 1: Utility is linear in consumption. In this case, the degree of risk aversion is extremely low (recall that risk aversion enters through the convexity of the search cost function). This makes our model and approach very similar to that of Mortensen (1994) and Millard and Mortensen (1995).

In fact, in this case, our model yields predictions that are almost identical to those in Mortensen (1994)—we find that the current UI program in the U.S. generates a dead weight loss of roughly 1.2 percent of welfare.<sup>15</sup>

With this low level of risk aversion, we find that the optimal UI program entails no benefits at all! That is, when the degree of risk aversion is low, the job destruction effect of UI is large enough to outweigh the positive impact of any insurance at all.<sup>16</sup> Clearly, this result depends upon the fact that when utility is linear in consumption the demand for unemployment insurance is low.

Case 2:  $U(C) = \ln(C)$ . In this case the Arrow-Pratt measure of relative risk aversion is constant and equal to one. This is the utility function used by Baily (1978) and is probably the utility function that is most often used in the literature on decision making under uncertainty. With these preferences, we obtain very different results. First, in stark contrast to the results obtained with linear utility, we find that the current U.S. unemployment insurance system (a 50 percent replacement rate for 26 weeks) increases welfare above the level that would be achieved without publicly provided UI. Moreover, the welfare gains are far from trivial—our estimate is that welfare rises by 1.2 percent.<sup>17</sup>

Our second set of results concerns the optimal UI program. Unfortunately, without savings, this utility function yields an unappealing result—the optimal potential duration of benefits must be infinity. The reason is simple. Since an unemployed worker who exhausts benefits earns no income,

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<sup>15</sup>The fact that we obtain results that are so similar to Mortensen (1994) despite differences in our data and approach to calibration is comforting. In addition, recall that the abbreviated model of sub-sections 3.A and 3.B yielded results remarkably similar to those found in Baily (1978) and Flemming (1978). Thus, our model seems to be able to reproduce the existing results in the literature once the assumptions are altered to match the models used by previous authors. This is true in spite of the fact that virtually all of the previous models were calibrated in different ways using data from a wide variety of different sources.

<sup>16</sup>This result holds for all of the parameter values that we consider.

<sup>17</sup>This is for our reference case.

that worker's instantaneous utility is  $\ln(0) = -\infty$ . Thus, if there is even the slightest probability of an unemployed agent exhausting her benefits, it follows that all agents' expected lifetime utility is  $-\infty$ . Any optimal UI program must therefore offer benefits indefinitely. This is clearly a knife-edge result—if UI exhaustees could count on even \$1 of income from friends or relatives while unemployed, this result would vanish. In fact, if we assume that UI exhaustees do obtain a \$1 transfer from family while unemployed, then we find that the job destruction effect does overturn the result that benefits

should be offered indefinitely, even with this high level of risk aversion. Nevertheless, the optimal value of T remains quite large—90 weeks in our reference case and from 74 to 104 weeks in the other cases

Table 1. Optimal UI Programs for Log Utility

	Potential Duration	Replacement Rate
Reference Case* s = .010, F = 96.25	90 weeks	64%
Few Firms s = .010, F = 95	84 weeks	64%
Many Firms s = .010, F = 97.5	94 weeks	64%
Low Turnover s = .007, F = 96.25	74 weeks	68%
High Turnover s = .013, F = 96.25	104 weeks	61%

\*In each case F (total demand for labor) is endogenous with parameters R (firm revenue) and K (the cost of maintaining a vacancy) chosen so that F matches the values listed. s denotes the bi-weekly separation rate.

(see Table 1)—so that benefits should be offered for as long as two full years. Thus, the job destruction effect is not nearly as important when agents are reasonably risk averse. As for the optimal replacement rate, when agents cannot self-insure by saving, the optimal replacement rate is about 64 percent. With savings, this rate is likely to fall below 50 percent (see section 5 below). We conclude that with reasonable assumptions concerning risk aversion, the optimal UI program offers benefits somewhat below 50 percent for almost two years. Our model predicts that instituting such

a UI program would raise welfare above the level achieved with the current U.S. program by 5.5 percent of welfare—a startlingly high measure for a potential welfare gain.

Case 3:  $U(C) = \sqrt{C}$ . This utility function has a constant Arrow-Pratt measure of relative risk aversion equal to 1/2, so that it falls mid-way between our other two utility functions. With this utility function we find that the current UI program in the US is just about right—the optimal program involves offering a replacement rate of 61 percent for 26 weeks in our reference case with programs of similar duration and replacement rates in all other cases (see Table 2). We also find that this optimal program increases welfare above the levels that would be achieved without a UI program by about 2 percent.

The differences in our three sets of results indicate that the assumptions made concerning risk aversion are crucial. Thus, it is important to determine which utility function best represents workers’ degree of risk aversion. To answer this question, there are two contradictory strands of literature that we may consult. First,

**Table 2. Optimal UI Programs for Square Root Utility**

	Potential Duration	Replacement Rate
Reference Case* s = .010, F = 96.25	26 weeks	61%
Few Firms s = .010, F = 95	24 weeks	61%
Many Firms s = .010, F = 97.5	28 weeks	61%
Low Turnover s = .007, F = 96.25	20 weeks	66%
High Turnover s = .013, F = 96.25	32 weeks	56%

\*In each case F (total demand for labor) is endogenous with parameters R (firm revenue) and K (the cost of maintaining a vacancy) chosen so that F matches the values listed. s denotes the bi-weekly separation rate.

there is the empirical literature on consumption behavior that attempts to estimate directly agents’ degree of risk aversion (see, for example, Zeldes 1989). This work suggests that the best point estimate of the Arrow-Pratt measure of relative risk aversion is 2. The other literature, which is theoretical, attempts to infer risk aversion from observed behavior. For example, we can observe

how agents adjust their investment portfolios as their wealth changes and we can build models of investment under uncertainty to explain such behavior. Most work in this area finds that the theories of choice under uncertainty are consistent with observed behavior only if the Arrow-Pratt measure of relative risk aversion is less than one (see, for example, Hadar and Seo 1993).

The fact that these two literatures contradict one another is troubling and leaves us (and economists generally) in an uncomfortable position. Our work indicates that if the Arrow-Pratt measure of relative risk aversion is close to (or above) one, then the current UI program in the US is not nearly generous enough. However, if the Arrow-Pratt measure of relative risk aversion is close to one-half, then the current system is about right. If one finds the empirical literature on consumption convincing (as we tend to), then the former outcome is more likely than the latter. Thus, we conclude that in the most general model with the most reasonable assumption concerning risk aversion, the optimal UI program offers benefits that are close to the levels currently offered by most states in the U.S. but offers those benefits for a much longer period of time—between 1.5 and 2 years. In other words, the current U.S. program does not offer enough unemployment insurance.

#### D. Discussion

The finding that the optimal UI program is characterized by fairly a low replacement rate and a long potential duration of benefits conflicts with most of the previous literature. However, we argue below that our results should have been expected, since they are consistent with the large abstract literature on optimal insurance contracts in the presence of moral hazard. We now offer a brief review of this literature since doing so allows us to view UI in a way that makes clear the economic sense behind our results.



Three issues have been addressed in the abstract literature on optimal insurance contracts that have implications for the design of an optimal UI program. The first issue concerns the design of an optimal insurance contract when the insured agent's behavior can effect the probability of a loss occurring (i.e., moral hazard is present). To investigate this issue, it is assumed that the agent's behavior cannot be observed by the insurance provider so that the contract must be structured in a manner that makes putting forth effort optimal for the agent. The problem then is how to provide adequate insurance without reducing the agent's incentive to avoid the loss. Shavell (1979) is a well known article in this area.<sup>18</sup>

The second issue concerns the optimal way to share risk between a risk neutral insurance provider and a risk averse agent when the total level of insurance coverage is fixed (i.e., in terms of expected indemnification). Raviv (1979) provides the classic treatment of this issue (see also Arrow 1974).

The final issue concerns the design of insurance contracts in the presence of adverse selection—the situation in which agents differ in a way that affects the probability that they will suffer a loss but that is unobservable to the insurers. The main problem in this case is to devise insurance contracts that will lead agents to self-select into groups and therefore reveal their personal characteristics. The classic article on this topic is Rothschild and Stiglitz (1976).

The remarkable thing about these strands of literature is that, although they ask different questions, in at least the simplest models they all deduce the same answer—optimal insurance contracts take the form of a “deductible policy” in which coverage is not provided for losses below

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<sup>18</sup>The parallel between garden variety insurance, such as fire insurance or automobile insurance, and UI is incomplete because moral hazard arises in a different way in each. With fire insurance, moral hazard occurs through lack of care in preventing a single event that results in a single insurance payment. With UI, it could be argued that moral hazard occurs mainly after a precipitating event—involuntary job loss—through lack of effort to gain reemployment. The lack of effort to gain reemployment results in a stream of insurance payments.

a certain level.<sup>19</sup> The reasoning is as follows. When agents face uncertainty in income they would like to smooth income as much as possible by purchasing insurance. In the absence of moral hazard, the optimal insurance contract in a competitive insurance market provides full coverage so that income is the same in all circumstances. However, when moral hazard is present, the market breaks down when full insurance is provided since; in that case, no agent would have any incentive to take care to avoid large losses. With no one taking care, large losses would occur and insurance providers would go broke compensating the insured. Thus, given that full insurance will not be provided, what type of insurance is best? To answer this, note that agents are most concerned about avoiding catastrophes—that is, extremely large losses. It follows that the outcomes that they are most concerned about insuring against are the most adverse outcomes, and any optimal insurance contract will have to provide coverage in such cases. The insurance contract must also provide incentives to avoid losses, and this is provided by not covering small losses—there is a deductible that the insured agent must cover any time a loss occurs. In summary, a deductible contract forces agents to cover all small losses and provides coverage against large losses. It is optimal since it provides coverage in the cases that agents are most concerned about and includes incentives for agents to put forth effort to avoid losses.

What are the implications of this literature for UI? For unemployed workers, large losses occur when they suffer long spells of unemployment. Thus, an optimal UI program should provide compensation to those who have a particularly difficult time finding reemployment. This is why we find that a long potential duration of benefits is optimal. As for the deductible, it has been ruled out in our model by requiring the replacement rate to remain constant over the spell of unemployment

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<sup>19</sup>In the case of adverse selection it is necessary to offer a menu of deductible policies so that agents with different characteristics choose different policies. However, each of the offered policies includes a non-zero deductible.

until benefits are exhausted.<sup>20</sup> Therefore, the only way to force agents to search for employment is to keep the replacement rate relatively low. This explains why our model yields optimal replacement rates at or below the current rates offered in the U.S.

The results from the optimal insurance literature also imply that the current UI program in the U.S. is reverse of what it should be. By offering benefits to most workers for 26 of the first 27 weeks of unemployment, the UI system covers the majority of short spells of unemployment—that is, the small losses. By cutting off benefits after 26 weeks of payments, the government provides inadequate coverage in catastrophic cases—ones in which agents suffer large losses due to long spells of unemployment.

The type of program that we are suggesting—one with low benefits offered for a long period of time—has the additional benefit that it would reduce the subsidization of temporary layoffs by firms. It is well-known that since the payroll tax that funds UI is incompletely experience rated in the U.S. (and not experience rated at all in other countries), firms have an incentive to exploit the system by temporarily laying off workers and then recalling them when their benefits expire. Existing estimates suggest that 25 to 50 percent of all layoffs in the U.S. can be explained by incomplete experience rating of the payroll tax (Card and Levine 1994; Topel 1994). If laid-off workers received lower benefits during the early weeks of unemployment, they would have an incentive to seek new jobs rather than wait for recall. And if workers are unwilling to wait for recall, then firms would be less likely to lay them off.<sup>21</sup>

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<sup>20</sup>This requirement also rules out a deductible in the form of a waiting period, which is a common feature of UI in the United States.

<sup>21</sup>Previous results have hinted that such a program might be more efficient than current UI programs. For example, O’Leary (1994) used a consumer theoretic approach to examine UI benefit adequacy and found that the current U.S. system over-compensates short spells of unemployment and under-compensates long spells. This conclusion is consistent with the type of policy shift suggested above. In addition, using a very different framework, Wang and Williamson (1996) have recently argued for a UI program that offers benefits for over 5 quarters! The government

## 5. EXTENSIONS

Up to this point, we have assumed that workers are not allowed to self-insure by saving while employed and that all workers are identical. In this section, we offer extended versions of our model to see how the results would be altered by relaxing these assumptions.

### A. Savings

Allowing workers to save while employed should reduce the generosity of the optimal UI program since workers will be able to supplement UI benefits by dissaving during the early stages of a jobless spell. For workers who suffer long spells of unemployment, however, savings will eventually be depleted, leaving them with only their UI benefits to finance consumption.

Extending our model to allow for savings is non-trivial. To do so in a rigorous way would require us to model the consumption/savings decision faced by each worker in each stage of employment and unemployment. Optimal consumption would depend not just on a worker's current job market status, but on his complete labor market history. This would require us to keep track of each worker's employment history and the economy-wide distribution of assets. Such an extension is clearly beyond the scope of this paper.

Instead, we make use of the recent empirical findings of Gruber (1997) who used data on food consumption from the Panel Study of Income Dynamics to analyze the consumption-smoothing benefits of UI. Gruber's sample consisted of individuals who were employed at the time of an annual interview in year  $t-1$  and then unemployed at the time of the interview in year  $t$ . He compared the

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is given much more power in their model—the benefit rate is allowed to vary over the spell of unemployment, the government is allowed to control consumption, and the government can offer reemployment bonuses to workers. Nevertheless, they find that it is optimal to offer positive benefits for over one year.

yearly consumption levels by these individuals and concluded that with a UI replacement rate of 50 percent, consumption in year  $t$  would be 94 percent of consumption in year  $t-1$ . In addition, he estimated that a replacement rate of roughly 80 percent would fully smooth consumption across the unemployment spell. Gruber's conclusion is that UI does allow workers to smooth consumption. His findings also indicate that personal savings alone are not sufficient to smooth consumption.

Our goal in this section is to make use of Gruber's findings to get some idea of how our results would be altered if agents saved while employed. To do so, we make a few simplifying assumptions. First, we assume that a worker who was employed at time  $t-1$  was employed for the full year, whereas a worker who was unemployed at time  $t$  suffered one spell of unemployment during the year and that the spell was of average duration (one quarter). Second, we assume that while employed, all agents save a constant fraction of their income (denoted by  $\sigma$ ) and that these savings are then used to finance consumption by unemployed workers not receiving UI (UI-exhaustees and UI-ineligibles) and to supplement benefits for those who are collecting UI. Moreover, we assume that the amount of savings transferred to those receiving benefits is independent of the number of weeks the worker has been unemployed and that the amount transferred to UI-exhaustees and UI-ineligibles is the same. To make this precise, let  $C_w$  denote consumption by an employed worker, let  $C_u$  represent consumption by an unemployed worker who is collecting UI-benefits, and let  $C_x$  denote consumption by UI-exhaustees and UI-ineligibles. Then we have  $C_w = [w(1 - \tau) + \theta_w](1 - \sigma)$ . Furthermore, for  $C_w$  and  $C_u$  to be consistent with Gruber's findings, it must be the case that when the replacement rate is 50 percent (as in Gruber's sample), then

$$.25C_u + .75C_w = .94C_w$$

(since a worker who is unemployed at time  $t$  is assumed to have been unemployed for one-quarter and employed for three quarters). This implies that if the replacement rate is 50 percent then  $C_u = .76C_w$ —that is, while unemployed a worker collecting UI benefits consumes 76 percent as much as an employed worker. Most of this consumption is covered by UI benefits while the remainder is paid for by savings.

To derive the optimal UI program, we need to know how consumption varies with the replacement rate. We can derive such a formula by making use of Gruber's estimate that a replacement rate of roughly 80 percent would allow unemployed workers to fully smooth consumption across the jobless spell. This implies that  $C_u = C_w$  when the replacement rate is .80. Combining these two results gives us the following relationship between  $C_u$ ,  $C_w$ , and RR:

$$(26) \quad C_u = (.36 + .8RR)C_w.$$

For UI-ineligibles and UI-exhaustees, we use one more estimate from Gruber. He estimates that with a replacement rate of zero, consumption in year  $t$  would be approximately 78 percent of what it had been in year  $t-1$ . For UI-ineligibles who are unemployed at time  $t$ ,  $C_x$  and  $C_w$  should therefore satisfy  $.25C_x + .75C_w = .78C_w$ . It follows that  $C_x = .12C_w$ —that is, while unemployed, consumption by UI-ineligibles and UI-exhaustees falls to 12 percent of consumption while employed. This consumption is fully financed by savings.

To complete the extended model we need to make sure that all savings are accounted for. Since there are  $J$  employed workers who each save  $[w(1 - \tau) + \theta_w]\sigma$ , total savings are  $J[w(1 - \tau) + \theta_w]\sigma$ . These savings are used to finance consumption by the unemployed. UI-eligibles who have not

exhausted their benefits receive  $C_u - x$ , while UI-ineligibles and UI-exhaustees receive  $C_x$ . Thus, for total savings to equal total payments we must have

$$(27) \quad J[w(1 - \tau) + \theta_w]\sigma = (U - U_x - U_i)(C_u - x) + (U_x + U_i)C_x.$$

With (26), (27), and the equation  $C_x = .12C_w$  added to our model we are now in a position to see how savings affect our results. To do so, we first recalibrate the model for the square root and log utility functions to get estimates of the parameters that make the model's predictions consistent with the Illinois Reemployment Bonus Experiment. We then solve for the optimal UI program in the reference case for each of the two utility functions.

As expected, in each case the optimal UI program is less generous than it would be in the absence of savings. Self-insurance lowers the optimal replacement rate by a significant amount and reduces the optimal potential duration of benefits as well. When the degree of relative risk aversion is one-half (the square root utility function), the optimal UI program is characterized by a replacement rate of 28 percent offered for 24 weeks. If we compare this with the optimal program without savings (RR = 61%, T = 26), we see that the main impact of self-insurance is to reduce the optimal replacement rate. When the degree of relative risk aversion is equal to one (the log utility function), the optimal UI program in the reference case is characterized by a replacement rate of 46 percent and a potential duration of benefits of 44 weeks. Thus, with log utility the ability to self-insure causes both the replacement rate and the potential duration of benefits fall significantly. Nonetheless, in both cases our main result with respect to potential duration remains—with a square root utility function the current system (with potential benefit duration of 26 weeks) is about right while with the log utility function the current system does not offer benefits for a long enough period of time. Both

models suggest that current benefit rates are too high, although the model that is closest in terms of the empirical estimates of risk aversion (the log model) yields a replacement rate quite close to current rates.

## B. Worker Heterogeneity

All of the previous work on UI, including our own, assumes that all agents are alike. In reality, however, workers are subject to a wide variety of labor market experiences. Some workers are never unemployed, others find jobs quickly, and some always face long spells of unemployment upon losing a job. In addition, some agents may attempt to take advantage of the UI system while others would never even consider claiming benefits. This implies that agents will have different preferences concerning unemployment insurance based on their labor market histories and expectations. Moreover, the number of workers that attempt to exploit the system may depend upon the generosity of the program.

In order to take worker heterogeneity into account, we extend our model to allow for three different classes of workers. The first class represents the bulk of the labor force and is described by the model introduced above. These workers face employment risk, losing their jobs with probability  $s$  in each period, and actively search for a new job once unemployed.

The second class consists of workers who are never unemployed. We refer to this group as “professionals” and use  $\phi$  to denote the proportion of the labor force that falls into this class. We also use  $L_p$  to denote the number of such workers and  $V_p$  to denote their expected lifetime utility. Since these workers are never unemployed, they earn  $w$  in each period of life, and thus,  $V_p =$



$u(w)(1+r)/r$ . The total contribution of these workers to social welfare is therefore  $L_p V_p$  and adding professionals to the model is accomplished by adding this term to  $W$  as defined in equation (4).

The last class of workers consists of agents who try to take advantage of the system. We refer to such workers as “opportunists.” We assume that these agents work only to become eligible for UI and that they live off of the dole as much as possible. We use  $L_o$  to denote the number of opportunists and use  $V_o$  to represent their expected lifetime utility. Thus, their contribution to social welfare ( $W$ ) is  $L_o V_o$ .

Presumably, the number of opportunists in the labor force will be a function of the generosity of the system—a more generous UI program should result in more opportunists. To measure the generosity of the system, we introduce the following variable  $G$ :

$$G = \{u(x)/u(w)\} \{1 - (1/1+r)^T\}.$$

$G$  measures the ratio of utility received from collecting UI benefits to utility from working for wage  $w$  during one spell of unemployment that lasts  $T$  periods (the potential duration of benefits). Note that if  $x = 0$  or  $T = 0$ , so that no UI is offered,  $G = 0$ . On the other hand, as the replacement rate approaches one and  $T$  approaches infinity,  $G$  approaches 1. Increases in  $G$  represent increases in the generosity of the UI program. We assume that  $\alpha$ , the proportion of the labor force that are opportunists, is positively related to  $G$ . In particular, we assume that  $\alpha = \eta G$ .

To complete the extended model, we must describe the determination of  $\eta$  and  $V_o$ . Consider  $V_o$  first. We assume that, since these agents work as little as possible, they contribute less to social welfare than the average unemployed agent (who, after all, is at least seeking a job). Thus, since  $V_u$  is the average expected lifetime utility for unemployed workers, we set  $V_o = \Omega V_u$  with  $\Omega < 1$ . We then

vary  $\Omega$  and see how this affects the optimal UI program. Note that  $\Omega$  measures the relative contribution of opportunists to social welfare.

For  $\eta$  (the extent to which opportunists appear in response to increases in generosity), we solve the model assuming that the current U.S. program is in effect (a 50 percent replacement rate offered for 26 weeks) and then vary  $\eta$  so that  $\alpha$  ranges from 0 to .05. Thus, we consider values for  $\eta$  implying that between 0 percent and 5 percent of the labor force is currently exploiting the system.

Our results for the square-root utility function are summarized in Tables 3 and 4 and our results for the log utility function are reported in Tables 5 and 6. In these tables, we report the optimal UI program for various values of  $\alpha$ ,  $\Omega$ , and  $\phi$  with all other parameters set at the reference case values. In each cell, the optimal UI program is reported by first listing the optimal replacement rate and then listing the optimal potential duration of benefits. Tables 3 and 5 show how the optimal UI program varies with  $\alpha$  and  $\Omega$  when there are no professionals in the model (i.e.,  $\phi = 0$ ). For example, if we are using the square-root utility function and  $\alpha = 0$ , so that there are no opportunists in the model, the optimal program offers a 61 percent replacement rate for 26 weeks. As the number

Table 3. Optimal UI Programs with Opportunists but No Professionals Using Square-root Utility (replacement rate, potential duration in weeks)

$\Omega$	$\alpha = 0$	.01	.02	.05
.9	(61%, 26)	(59%, 24)	(56%, 22)	(48%, 16)
.8	(61%, 26)	(60%, 22)	(55%, 20)	(42%, 14)
.7	(61%, 26)	(58%, 22)	(54%, 18)	(36%, 12)

$\alpha$  is the proportion of the labor force comprised of opportunists.  
 $\Omega$  is the relative contribution of opportunists to social welfare.

of opportunists increases, the generosity of the optimal program declines regardless of the value of  $\Omega$ .

This is hardly surprising—with more opportunists in the economy the government needs to make the

program less generous in order to discourage exploitation of the system.

Tables 3 and 5 also indicate that the generosity of the optimal program is decreasing in  $\Omega$ , the parameter that measures the amount that opportunists contribute to social welfare. As  $\Omega$  decreases, opportunists contribute less to social welfare and it becomes more important

Table 4. Optimal UI Programs with Professionals and Opportunists Using Square-root Utility (replacement rate, potential duration in weeks)

$\phi$	$\alpha = 0$	.01	.02	.05
0	(61%, 26)	(60%, 22)	(55%, 20)	(42%, 14)
.1	(64%, 28)	(60%, 26)	(57%, 22)	(45%, 14)
.2	(66%, 32)	(63%, 28)	(61%, 24)	(47%, 16)
.3	(68%, 36)	(67%, 32)	(64%, 28)	(51%, 18)

$\alpha$  is the proportion of the labor force comprised of opportunists.  
 $\phi$  is the proportion of the labor force comprised of professionals.  
 $\Omega$  is the relative contribution of opportunists to social welfare.  
 $\Omega = .8$  in all cells.

for the government to discourage exploitation. Tables 3 and 5 clearly indicate the importance of the actual values of  $\alpha$  and  $\Omega$ . If  $\alpha$  is low or if  $\Omega$  is close to one, then the optimal program is quite close to the optimal program in the model that ignores exploitation. But for large values of  $\alpha$  and low values of  $\Omega$  (e.g.,  $\alpha = .05$  and  $\Omega = .7$ ), the optimal program is considerably less generous.

Table 5. Optimal UI Programs with Opportunists but No Professionals Using Log Utility (replacement rate, potential duration in weeks)

$\Omega$	$\alpha = 0$	.01	.02	.05
.9	(64%, 90)	(62%, 86)	(60%, 82)	(53%, 76)
.8	(64%, 90)	(62%, 84)	(59%, 78)	(47%, 70)
.7	(64%, 90)	(61%, 82)	(56%, 74)	(40%, 66)

$\alpha$  is the proportion of the labor force comprised of opportunists.  
 $\Omega$  is the relative contribution of opportunists to social welfare.

Tables 4 and 6 report the optimal program when opportunists and professionals are included in the model. These results are derived assuming that  $\Omega = .8$  (as in the middle row of Tables 3 and 5). Tables 4 and

6 indicate that as the proportion of professionals rises the optimal program becomes more generous. The reasoning is as follows. With professionals in the model, the tax burden of UI is shared by a group of workers who never use the system, which allows for a more generous system. As in Tables 3 and 5, knowing the correct value of  $\phi$  is important. For example, when we use the square-root

utility function and set  $\alpha = 0$  we find that the optimal UI program when 10 percent of the work force is made up of professionals offers a replacement rate of 64 percent for 28 weeks. If, on the other hand, 30 percent of the work force are professionals, the optimal program

**Table 6. Optimal UI Programs with Professionals and Opportunists Using Log Utility (replacement rate, potential duration in weeks)**

$\phi$	$\alpha = 0$	.01	.02	.05
0	(64%, 90)	(62%, 84)	(58%, 74)	(48%, 60)
.1	(66%, 94)	(65%, 88)	(61%, 77)	(51%, 62)
.2	(68%, 100)	(67%, 92)	(64%, 80)	(55%, 66)
.3	(70%, 102)	(68%, 96)	(66%, 84)	(58%, 70)

$\alpha$  is the proportion of the labor force comprised of opportunists.  
 $\phi$  is the proportion of the labor force comprised of professionals.  
 $\Omega$  is the relative contribution of opportunists to social welfare.  
 $\Omega = .8$  in all cells.

offers a replacement rate slightly higher (68 percent) but for a much longer time (36 weeks).

We conclude that our main result holds up to extensions that allow for saving by workers and worker heterogeneity. Although the approaches taken to modeling savings and worker heterogeneity are relatively simple, we believe that they correctly point toward the answers that could be expected if more comprehensive approaches were taken. Regarding savings, the approach taken is consistent with the limited empirical work on this topic, however simple the approach may be. Regarding worker heterogeneity, although we have added two new groups to the model, we have not modeled transitions between the groups (although the number of opportunists does depend on the generosity of the program in the model). To some extent, efforts in these directions may necessarily be limited until we know more about the composition of the labor force (that is, the proportions of workers that can be considered “opportunists,” “professionals,” and so on) and the extent to which workers may move among various labor force groups. So although we believe that the answers provided above are broadly correct, there is clearly room for additional work on optimal UI when workers are heterogeneous and can save.

## 6. CONCLUSION

We have presented a general equilibrium search model of the labor market in order to determine the optimal UI program when the government sets the optimal potential duration of benefits and the replacement rate must remain constant over the spell of unemployment until benefits are exhausted. The model we develop has several features that have not been included in earlier work on optimal UI. First, it is an equilibrium model that allows us to measure the costs of different UI programs through the impact on search effort, job creation, and unemployment. Second, we have assumed that workers are risk averse so that we can measure the welfare benefits of different UI programs through the insurance that they provide against the risk of unemployment. Third, our model allows the potential duration of benefits to vary, includes workers who are ineligible for UI, takes account of firm behavior so that we can measure the job destruction effects of UI, and allows for worker heterogeneity and private savings. As far as we know, the above features make this model the most comprehensive in the literature.

Our basic finding is that current statutory UI replacement rates offered in the U.S. are about right (or slightly too high), but that UI benefits are not offered for a long enough period of time. Thus, we conclude that the U.S. system is not generous enough. This finding depends heavily on the degree of worker risk aversion and it also depends on our assumption that all unemployed workers are alike. If workers differ significantly in their labor market experiences, the division of the labor force into professionals and opportunists becomes important as well. The result appears to be much less sensitive to assumptions about whether workers can save. Unfortunately, not much is known

about the degree of worker risk aversion or the division of the labor force into different groups.

Further empirical work aimed at clarifying these parameters would have high value.

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