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Researching the causes and consequences of unemployment

Aggregate Demand, Idle Time, and Unemployment

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ABSTRACT

This paper develops a model of unemployment fluctuations. The model keeps the architecture of the Barro and Grossman (1971) general disequilibrium model but replaces the disequilibrium framework on the labor and product markets by a matching framework. On the product and labor markets, both price and tightness adjust to equalize supply and demand. There is one more variable than equilibrium condition on each market, so we consider various price mechanisms to close the model, from completely flexible to completely rigid. With some price rigidity, aggregate demand influences unemployment through a simple mechanism: higher aggregate demand raises the probability that firms find customers, which reduces idle time for firms' employees and thus increases labor demand, which in turn reduces unemployment. We use the comparative-statistics predictions of the model together with empirical measures of quantities and tightnesses to re-examine the origins of labor market fluctuations. We conclude that (1) price and real wage are not fully flexible because product and labor market tightness fluctuate significantly; (2) fluctuations are mostly caused by labor demand and not labor supply shocks because employment is positively correlated with labor market tightness; and (3) labor demand shocks mostly reflect aggregate demand and not technology shocks because output is positively correlated with product market tightness..

JEL Classification Codes: E10, E24, E30, J2, J64

Key Words: aggregate demand, unemployment, matching frictions, business cycles

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Abstract

This paper develops a model of unemployment fluctuations. The model keeps the architecture of the Barro and Grossman [1971] general disequilibrium model but replaces the disequilibrium framework on the labor and product markets by a matching framework. On the product and labor markets, both price and tightness adjust to equalize supply and demand. There is one more variable than equilibrium condition on each market, so we consider various price mechanisms to close the model, from completely flexible to completely rigid. With some price rigidity, aggregate demand influences unemployment through a simple mechanism: higher aggregate demand raises the probability that firms find customers, which reduces idle time for firms' employees and thus increases labor demand, which in turn reduces unemployment. We use the comparative-statics predictions of the model together with empirical measures of quantities and tightnesses to re-examine the origins of labor market fluctuations. We conclude that (1) price and real wage are not fully flexible because product and labor market tightness fluctuate significantly; (2) fluctuations are mostly caused by labor demand and not labor supply shocks because employment is positively correlated with labor market tightness; and (3) labor demand shocks mostly reflect aggregate demand and not technology shocks because output is positively correlated with product market tightness.

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1 Introduction

The US unemployment rate did not fall below 7% from 2009 to 2013. The origins of this five-year period of high unemployment are still debated. Indeed, a large number of candidates have emerged to explain this period of high unemployment. Popular candidates include high mismatch, caused by major sectoral shocks, low search effort from unemployed workers, triggered by the long extensions of unemployment insurance benefits, and low aggregate demand, caused by a sudden need to repay debts or pessimism.[1] Low technology would be another natural candidate to explain high unemployment since technology shocks are the main source of fluctuations in the textbook model of unemployment.[2]

To explore the origins of unemployment fluctuations, two popular models could be used: a matching model of the labor market or a New Keynesian dynamic stochastic general equilibrium (DSGE) model. The matching model has many desirable properties.[3] It delivers useful comparative statics for a variety of labor market shocks, and it accurately represents the mechanics of the labor market. But the matching model completely abstracts from the influence of aggregate demand on the labor market; therefore, it leaves out a potentially important source of unemployment fluctuations. On the other hand, DSGE models capture the effect of aggregate demand and many other shocks on the labor market.[4] But these models have evolved towards greater complexity, making it difficult to characterize analytically the effects of these shocks on unemployment.

We therefore think that we need a third model, between a matching model and a DSGE model, that accounts for a number of factors of unemployment, including aggregate demand, and that lends itself to comparative-statics analysis. Some people have thought that the general disequilibrium model of Barro and Grossman [1971] could be the third model.[5] This model captures the link between aggregate demand and unemployment, but it is complicated to analyze as the economy

[1] On mismatch, see Sahin et al. [2014], Lazear and Spletzer [2012], and Diamond [2013]. On job-search effort, see Elsby, Hobijn and Sahin [2010], Rothstein [2011], and Farber and Valletta [2013]. On aggregate demand, see Farmer [2013, 2011] and Mian and Sufi [2012].

[2] See for instance Pissarides [2000] and Shimer [2005].

[3] See Pissarides [2000] for an overview.

[4] While unemployment is absent from standard DSGE models, new DSGE models have been developed to incorporate it. See Galí [2010] for an overview.

[5] The macroeconomic implications of general disequilibrium theory are discussed in Barro and Grossman [1976] and Malinvaud [1977]. This theory generated a vast amount of research. For surveys of this literature, see Grandmont [1977], Drazen [1980], and Bénassy [1993]. For recent applications of the theory, see Mankiw and Weinzierl [2011], Caballero and Farhi [2014], and Korinek and Simsek [2014].

can be in four possible regimes, depending on which side of each market is rationed, and these regimes are described by different systems of equations. The situation of disequilibrium also raises questions about how to ration the side of the market that cannot buy or sell what it planned to.

This paper proposes a model that, we think, addresses the limitations of the Barro-Grossman model. Our model can be seen as an equilibrium version of the Barro-Grossman model. It keeps the architecture of the Barro-Grossman model but replaces the disequilibrium framework on the product and labor markets by a matching framework.[6] The model is static. There are three goods: a nonproduced good, a produced good, and labor. The market for nonproduced good is perfectly competitive. The product and labor markets have matching frictions. This means that the number of trades is governed by a matching function taking as arguments the number of goods for sale and aggregate buying effort, and that buyers incur a matching cost per unit of effort. From a seller's perspective, a frictional market looks like a competitive market except that she takes into account not only the price but also the selling probability, which is less than one. From a buyer's perspective, a frictional market looks like a competitive market except that she takes into account the effective price of consumption, which is the market price times a wedge that captures the cost of matching. The selling probability and the matching wedge are determined by the market tightness.

An equilibrium is a set of prices and tightnesses such that supply equals demand on all markets. In each frictional market, there is one more variable than equilibrium condition; hence, many combinations of prices and tightnesses are consistent with the equilibrium. To close the model, we consider several price mechanisms. We focus on two polar mechanisms: (1) fixed prices, which are parameters of the model, and (2) efficient prices, which ensure that tightnesses are always efficient. We also show that all the comparative statics under fixed prices remain valid when prices are only partially rigid functions of the parameters, and all the comparative statics under efficient prices remain valid under Nash bargained prices.

Compared to the matching literature on unemployment, a contribution of our model is to explain how aggregate demand influences unemployment. It is therefore important to understand the mechanism, especially because it differs from the Barro-Grossman mechanism.[7] When prices are

[6]For other models with matching frictions on the product market, see Gourio and Rudanko [2014] and Bai, Rios-Rull and Storesletten [2012]. For other models with matching frictions on both the product and the labor market, see Hall [2008], Lehmann and Van der Linden [2010], and Petrosky-Nadeau and Wasmer [2011].

[7]In the Barro-Grossman model, the rigid price and aggregate demand determine the output produced by firms. If the employment required by firms to produce this fixed level of output is below the fixed labor supply, there is

completely flexible, aggregate demand has no effect on unemployment. When prices are partially rigid, aggregate demand influences unemployment as follows: higher aggregate demand raises product market tightness in equilibrium; this rise increases the probability that firms find customers, reduces idle time for firms' employees, and thus increases labor demand; finally, higher labor demand raises labor market tightness and reduces unemployment in equilibrium.

The model is tractable enough to obtain comparative statics with respect to a broad set of shocks—labor supply shocks, mismatch shocks, technology shocks, or aggregate demand shocks—under several price mechanisms. To identify the sources of unemployment fluctuations in the data, we compare the correlations arising from comparative statics with the corresponding correlations measured in the data.[8] While series for employment, output, and labor market tightness are readily available, a series for product market tightness is not. Hence, we construct a proxy for product market tightness based on the series of capacity utilization from the Survey of Plant Capacity of the Census Bureau. Three conclusions emerge from the empirical analysis. First, price and real wage are not fully flexible because product and labor market tightness fluctuate significantly. Second, employment fluctuations are mostly caused by labor demand and not labor supply shocks because employment is positively correlated with labor market tightness. Third, labor demand shocks mostly reflect aggregate demand and not technology shocks because output is positively correlated with product market tightness.

Our results are consonant with those obtained by other researchers, but as far as we know, it is the first time that the results are obtained through an integrated analysis. Our first result is related to the finding of Shimer [2005] and Hall [2005] that the observed fluctuations in labor market tightness indicate some real wage rigidity. Our second result is related to the finding of Blanchard and Diamond [1989*b*] that labor demand shocks are the main source of unemployment fluctuations. Our third result is related to the findings of Galí [1999] and Basu, Fernald and Kimball [2006] that technology shocks do not explain a large share of macroeconomic fluctuations.

unemployment and unemployment depends of course on the level of aggregate demand.

[8]This approach is similar in spirit to that followed by Blanchard and Diamond [1989*b*].

2 A Basic Model of Aggregate Demand and Idle Time

This section presents a simplified version of the model of Section 3, which is the main model of the paper. In this section we abstract from the labor market by assuming that all production takes place within the households, and not within firms. We thus simplify the presentations of the matching frictions on the product market and of the equilibrium, which are the two most important new elements of the framework. This section also provides empirical evidence in support of matching frictions on the product market.

2.1 Assumptions

The model is static. The assumption that the model is static will be relaxed in Section 4. There are two goods: a produced good and a nonproduced good. The economy is composed of a measure 1 of households.

The Market for Nonproduced Good. Each household has an endowment $\mu > 0$ of nonproduced good. The nonproduced good acts as numeraire. Households trade this good on a perfectly competitive market. The nonproduced good enter households' utility function. Hence, households allocate their income between consumptions of produced and nonproduced goods as a function of the relative price of the goods. The optimal allocation will determine aggregate demand.

The Product Market. Households act as firms in that they directly produce the produced good. The capacity of each household is k; that is, a household is able to produce k units of the good. Households are also consumers of produced good, but they cannot consume their own production. Each household visits v other households to purchase their production. The number of trades y on the product market is given by a matching function with constant returns to scale. For concreteness, we assume that the matching function takes the form

$$y = \left(k^{-\gamma} + v^{-\gamma}\right)^{-\frac{1}{\gamma}},$$

where $\gamma > 0$ determines the elasticity of substitution between inputs in the matching function. Since $\gamma > 0$, y is less than k and v.[9] In each trade, one unit of good is exchanged at price $p > 0$.

We define product market tightness as the ratio of visits to capacity: $x \equiv v/k$. With constant returns to scale in matching, product market tightness alone determines the probabilities to trade for sellers and buyers: one unit of produced good is sold with probability

$$f(x) = \frac{y}{k} = \left(1 + x^{-\gamma}\right)^{-\frac{1}{\gamma}}, \tag{1}$$

and one visit leads to a purchase with probability

$$q(x) = \frac{y}{v} = \left(1 + x^{\gamma}\right)^{-\frac{1}{\gamma}}. \tag{2}$$

The function f is smooth and strictly increasing on $[0, +\infty)$, with $f(0) = 0$ and $\lim_{x \to +\infty} f(x) = 1$. The function q is smooth and strictly decreasing on $[0, +\infty)$, with $q(0) = 1$ and $\lim_{x \to +\infty} q(x) = 0$. The properties of the derivative f' will also be useful. We can show that $f'(x) = q(x)^{1+\gamma}$; hence, f' is strictly decreasing on $[0, +\infty)$ with $f'(0) = 1$ and $\lim_{x \to +\infty} f'(x) = 0$. An implication is that f is strictly concave on $[0, +\infty)$. The properties of f and q mean that it is easier to sell goods but harder to buy them when the product market tightness is higher. Finally, we abstract from randomness at the household level: a household sells $f(x) \cdot k$ units and purchases $q(x) \cdot v$ units of good for sure.

We model matching costs as follows. Each visit requires to purchase $\rho \in (0, 1)$ units of produced good.[10] From the perspective of a buyer, this amount is dissipated and does not contribute to consumption c. The $\rho \cdot v$ units of good for matching are purchased like the c units of consumption. Hence, the number of visits is related to consumption and market tightness by $q(x) \cdot v = c + \rho \cdot v$. Therefore, the desired level of consumption determines the number of visits: $v = c / (q(x) - \rho)$. Because of matching costs, consuming one unit of produced good requires

[9] The matching function is borrowed from den Haan, Ramey and Watson [2000]. A required property for a matching function is that $y \leq \min\{k, v\}$. This matching function always satisfies it. We opt to use this matching function instead of the standard Cobb-Douglas matching function, $y = k^{\gamma} \cdot v^{1-\gamma}$, because the Cobb-Douglas matching function would need to be truncated to ensure that $y \leq \min\{k, v\}$, and the truncation would complicate the analysis.

[10] While many properties of the model would hold if $\rho = 0$, assuming $\rho > 0$ has several advantages. Among them, the equilibrium exists for any price level, and the efficient tightness is finite.

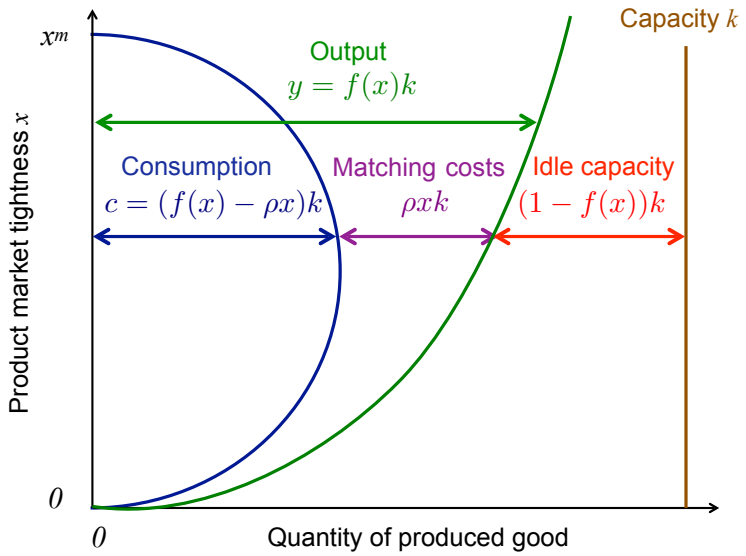


Figure 1: The Matching Frictions on the Product Market

buying $1 + (\rho \cdot v/c) = 1 + \tau(x)$ units of produced good, where

$$\tau(x) \equiv \frac{\rho}{q(x) - \rho}.$$

The function τ is positive and strictly increasing for all $x \in [0, x^m)$, where $x^m > 0$ is uniquely defined by $q(x^m) = \rho$. We also have $\tau(0) = \rho/(1-\rho)$ and $\lim_{x \to x^m} \tau(x) = +\infty$. Note that any equilibrium satisfies $x \in [0, x^m)$. At the limit where $\rho \to 0$, $\tau(x) \to 0$.

Figure 1 illustrates the matching frictions on the product market. It plots capacity, output, and consumption as a function of product market tightness. Output is $y = f(x) \cdot k$ so output is an increasing and concave function of tightness. Consumption is $c = f(x) \cdot k/(1 + \tau(x)) = (f(x) - \rho \cdot x) \cdot k$ so consumption first increases and then decreases with tightness.[11] A total of $\rho \cdot v = \rho \cdot x \cdot k$ units of produced goods are goods dissipated in matching. The gap between consumption and output represents the cost of matching. A total of $k - f(x) \cdot k = (1 - f(x)) \cdot k$ units of goods could have been produced during the time when workers are idle. The gap between output and capacity represents this idle capacity.

The consumption curve corresponds to the aggregate supply. We define the aggregate supply

[11] Given the definition of τ, $f(x)/(1+\tau(x)) = (f(x)/q(x)) \cdot (q(x) - \rho)$. By definition of f and q, $f(x)/q(x) = x$ and $x \cdot q(x) = f(x)$. Therefore, $f(x)/(1+\tau(x)) = f(x) - \rho \cdot x$.

as the amount of consumption traded for a given tightness, taking into account that households supply a fixed quantity k of produced goods to the market:

DEFINITION 1. *The* aggregate supply *is a function of product market tightness defined by*

$$c^s(x) = \frac{f(x)}{1+\tau(x)} \cdot k = (f(x) - \rho \cdot x) \cdot k \tag{3}$$

for all $x \in [0, x^m]$*, where* $x^m > 0$ *satisfies* $q(x^m) = \rho$*. The function* c^s *is strictly increasing on* $[0, x^*]$ *and strictly decreasing on* $[x^*, x^m]$*, where* $x^* \in (0, x^m)$ *is uniquely defined by* $f'(x^*) = \rho$*. We also have* $c^s(0) = 0$ *and* $c^s(x^m) = 0$*.*

The shape of the aggregate supply, first increasing then decreasing with x, is unusual, but it naturally arises from the properties of the matching function. When x is low, the matching process is congested by the amount of production for sale so increasing x—that is, increasing the number of visits relative to available production—leads to a large increase in the probability to sell, $f(x)$, but a small increase in the matching wedge faced by buyers, $\tau(x)$. Since the aggregate supply is proportional to $f(x)/(1+\tau(x))$, it increases. Conversely when x is high, the matching process is congested by the number of visits, and increasing x leads to a small increase in $f(x)$ but a large increase in $\tau(x)$ so an overall decrease in aggregate supply. The tightness x^* maximizes the aggregate supply.

Households. Households have a constant-elasticity-of-substitution utility function given by

$$\left(\frac{\chi}{1+\chi} \cdot c^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+\chi} \cdot m^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

where c is consumption of produced good, m is consumption of nonproduced good, $\chi \in (0, +\infty)$ is a parameter measuring the taste for produced good relative to nonproduced good, and ε is a parameter measuring the elasticity of substitution between produced good and nonproduced good. To guarantee the unicity of the equilibrium, we impose $\varepsilon > 1$.

We focus on consumption decisions and relegate the matching process to the background. Consuming c requires purchasing $(1+\tau(x)) \cdot c$ in the course of $(1+\tau(x)) \cdot c/q(x)$ visits, which costs a total of $p \cdot (1+\tau(x)) \cdot c$. In sum, the matching frictions imposes a wedge $\tau(x)$ on the price of

produced good.

The household's income comes from the sales of μ units of nonproduced good at price 1 and $f(x) \cdot k$ units of produced good at price p. The household uses the income to purchase m units of nonproduced good at price 1 and c units of produced good at price $(1+\tau(x)) \cdot p$. Hence, the household's budget constraint is

$$m + (1+\tau(x)) \cdot p \cdot c = \mu + p \cdot f(x) \cdot k. \tag{5}$$

Given x and p, the household chooses m and c to maximize (4) subject to (5). The optimal consumption satisfies

$$\frac{1}{1+\chi} \cdot (1+\tau(x)) \cdot p \cdot m^{-\frac{1}{\varepsilon}} = \frac{\chi}{1+\chi} \cdot c^{-\frac{1}{\varepsilon}}. \tag{6}$$

Equation (6) says that at the optimum, a household is indifferent between spending income on the nonproduced or the produced good. The aggregate demand gives the optimal consumption of produced good for a given tightness and price, taking into account the market-clearing condition on the market for nonproduced good ($m = \mu$):

DEFINITION 2. *The* aggregate demand *is a function of market tightness and price defined by*

$$c^d(x,p) = \frac{\chi^\varepsilon \cdot \mu}{(1+\tau(x))^\varepsilon \cdot p^\varepsilon} \tag{7}$$

for all $(x,p) \in [0,x^m] \times (0,+\infty)$*, where* $x^m > 0$ *satisfies* $\rho = q(x^m)$*. The function* c^d *is strictly decreasing in* x *and* p*. We also have* $c^d(0,p) = \chi^\varepsilon \cdot \mu \cdot (1-\rho)^\varepsilon / p^\varepsilon$ *and* $c^d(x^m,p) = 0$*.*

The aggregate demand c^d is strictly decreasing in x and p because the effective price of produced good is $(1+\tau(x)) \cdot p$ and an increase in effective price reduces the consumption of produced good relative to that of nonproduced good, fixed to μ.

2.2 Discussion of the Assumptions

We discuss two critical assumptions of the model: the presence of a nonproduced good, and the presence of matching frictions on the product market.

The assumption that households also consume a nonproduced good is borrowed from Barro

and Grossman [1971]. The nonproduced good is necessary to obtain an interesting concept of aggregate demand in a static environment, because without it, consumers would mechanically spend all their income on the produced good (Say's law). Here households allocate their income between consumptions of produced and nonproduced good, and aggregate demand is the desired consumption of produced good.[12] One can think of the nonproduced good as real money balances, gold or silver, land, or a fixed stock of capital.[13]

To represent matching frictions on the product market, we assume that the number of trades is governed by a matching function and that buyers face a matching cost. Here we argue that these assumptions realistically capture key aspects of the product market.

The Matching Function. In the same way as the production function summarizes how input are transformed into output through the production process, the matching function summarizes how available production and aggregate buying efforts are transformed into trades through a the matching process. The matching function provides a tractable representation of a very complex trading process. Its main implication is that not all available production is sold and not all visits by buyers are successful. Formally, we assume that households only sell a fraction $f(x) < 1$ of their production capacity and buyers' visits are only successful with probability $q(x) < 1$. The matching function is a useful modeling tool only if we find convincing evidence that at all times, some visits are unsuccessful and some capacity is idle.[14]

Visits are the product-market equivalent of vacancies. A visit represents the process that a buyer must follow to buy an item. These visits can take different forms, depending on the buyer. For a individual consumer, a visit may be an actual visit to a restaurant, to a hair salon, to a bakery, or to a car dealer. A visit could also be an inquiry to an intermediary, such as a travel agent, a real estate agent, or a stoke broker. For a firm, a visit could be an actual visit to a potential supplier. A visit could also be the preparation and processing of a request for proposal (RFP) or request for

[12] The representation of aggregate demand is quite abstract. Usually, we think that aggregate demand arises from a decision between consumption and savings in a dynamic environment. In Michaillat and Saez [2014], we extend the model in that direction. We allow households to save with money and bonds in a dynamic environment. We find that the properties of the aggregate demand and of the equilibrium remain broadly the same.

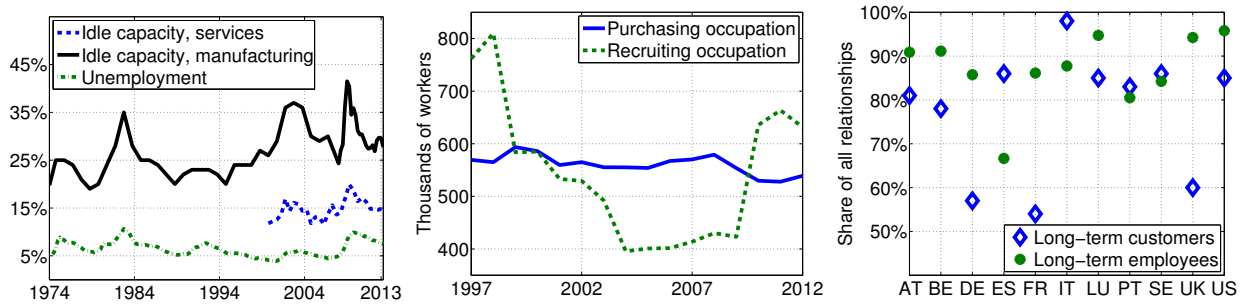
[13] Hart [1982] use a generic nonproduced good. Barro and Grossman [1971] use real money balances as a nonproduced good, as do Blanchard and Kiyotaki [1987].

[14] On the labor market, the concept of the matching function was pioneered by Pissarides [1985]. The labor market matching function became widely used after the empirical work of Pissarides [1986], Blanchard and Diamond [1989*a*], and others established that at all times, numerous vacancies are unfilled and numerous workers are unemployed.

tender (RFT) or any other sourcing process. Unlike for vacancies, however, visits are not recorded in any dataset, and it is not possible to provide quantitative evidence on the share of visits that are unsuccessful. Nevertheless, casual observation suggests that a significant share of visits do not generate a trade. At a restaurant, a consumer sometimes need to walk away because no tables are available or the queue is too long. The same may happen at a hair salon if no slots are available or if the salon is not open for business. At a bakery, the type of bread or cake desired by a consumer may not be available at the time of the visit, either because it was not prepared on the day or because the bakery has run out of it. At a car dealer, the specific car desired by the consumer may not be in inventory and may therefore not be available before a long time. For firms, buyers travel the world to assess the production plant or examine the quality of the work of potential supplier, and many of these visits do not lead to a contract. When a firm issues a RFP or RFT, it considers many applications from potential suppliers but only one is eventually selected.

The implication that not all production capacity is sold can be examined empirically. In the data, we find that idle capacity prevails at all time. Figure 2(a) displays the rate of idle capacity (one minus the rate of capacity utilization) in the US. For manufacturing, capacity utilization is measured by the Census Bureau from the Survey of Plant Capacity (SPC). It indicates the actual production level as a share of the production level that would maximize profits given the existing capital stock. For services, idle capacity is measured by the Institute for Supply Management (ISM). It indicates the actual production level as a share of the production level that would maximize profits given the existing capital stock and the existing workforce. On average the rate of idle capacity is 26.4% in manufacturing and 14.8% in services. In other words, on average , 26.4% of production plants are idle, 14.8% of the tables in restaurants and the chairs in hair salons are empty, and 14.8% of workers in consultancies and architecture firms are idle. For comparison, Figure 2(a) displays the unemployment rate constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). The rate of unemployment is the labor-market equivalent of the rate of idle capacity. The average unemployment rate is 6.5%, much lower than the average rate of idle capacity.

There is additional evidence that firms face difficulty in selling their production in the US. Using output and price microdata from the Census of Manufacturers conducted by the Census Bureau, Foster, Haltiwanger and Syverson [2012] find that at equivalent technical efficiency levels,



(a) Selling difficulties (b) Purchasing costs (c) Long-term relationships

Figure 2: Evidence of Matching Frictions on the Labor and Product Markets

Notes: In panel (a), idle capacity is 1 minus capacity utilization. Capacity utilization for manufacturing is measured for 1974–2013 by the Census Bureau from the SPC. Capacity utilization for services is measured for 1999–2013 by the ISM. The unemployment rate is constructed by BLS from the CPS for 1974–2013. In Panel (b), the number of workers in recruiting and purchasing occupations is from the OES database constructed by the BLS. Depending on the year, the occupation codes for purchasing occupations are 11-3060, 11-3061, and 13008 for purchasing managers; 13-1020, 13-1021–1023, and 21302–21308 for buyers and purchasing agents; and 43-4160, 43-4161, and 55314 for procurement clerks. Depending on the year, the occupation codes for recruiting occupations are 11-3120, 11-3049, 11-3040, and 13005 for human resource managers; 13-1071 and 21508–21511 for human resource specialists; and 43-3060, 43-3061, and 55326 for human resource assistants. In Panel (c), the share of relationships with repeat customers is from firm surveys in various countries: Kwapil, Baumgartner and Scharler [2005] sampled 873 firms in Austria (AT) in 2004; Aucremanne and Druant [2005] sampled 1,979 firms in Belgium (BE) in 2004; Stahl [2005] sampled 1,200 firms in Germany (DE) in 2004; Alvarez and Hernando [2005] sampled 2,008 firms in Spain (ES) in 2004; Loupias and Ricart [2004] sampled 1,662 firms in France (FR) in 2004; Fabiani, Gattulli and Sabbatini [2004] sampled 333 firms in Italy (IT) in 2003; Lunnemann and Matha [2006] sampled 367 firms in Luxembourg (LU) in 2004. Martins [2005] sampled 1,370 firms in Portugal (PT) in 2004; Apel, Friberg and Hallsten [2005] sampled 626 firms in Sweden (SE) in 2000; Hall, Walsh and Yates [2000] sampled 654 firms in the UK in 1995; and Blinder et al. [1998] sampled 200 firms in the US in 1990–1992. The share of workers in long-term employment relationships is from the OECD database for 2005.

new plants grow more slowly than established plants with a customer base because it is more difficult for them to sell their production. They conclude that despite similar or lower prices, new plants grow slowly because they need time to attract new customers.

The Matching Costs. A buyer faces a cost for a each visit. We make the crude but convenient assumption that this cost is incurred in the produced good. In reality, the buying cost takes many different forms. Of course, some costs are incurred in terms of produced good. When a consumer uses a travel agency to book a vacation, the cost of purchasing hospitality services is the price of the travel agent's services. When a consumer takes a cab ride to get to a hair salon, the cost of purchasing hairdressing services is the price of taxi services. When a firm hires a headhunter to recruit a manager, the cost of purchasing labor services is the price of executive search services.

But there are other types of cost too. For consumers, the cost of a visit could be the traveling

time required by the visit, or the time spent in a queue at a restaurant or hair salon. For firms, a large share of the cost of sourcing goods and services is a labor cost, and we are able to quantify this cost. Using data from the Occupational Employment Survey (OES) database constructed by the BLS, we compute the number of workers devoted to sourcing goods and services in US firms. The classification of occupations evolves from year to year so it is impossible to be completely consistent, and comparisons across years are not very meaningful. We measure the number of workers whose occupation is in buying, purchasing, and procurement. On average, 560,600 workers were employed in such occupations between 1997 and 2012. For comparison, Figure 2(b) displays the number of workers in human resources devoted to recruiting—recruitment, placement, screening, and interviewing. On average 543,200 workers were employed in these occupations from 1997 to 2012. Hence, the numbers of buyers and recruiters have the same order of magnitude.

2.3 Definition and Illustration of the Equilibrium

DEFINITION 3. *An equilibrium consists of a product market tightness and a price* (x, p) *such that aggregate supply is equal to aggregate demand:*

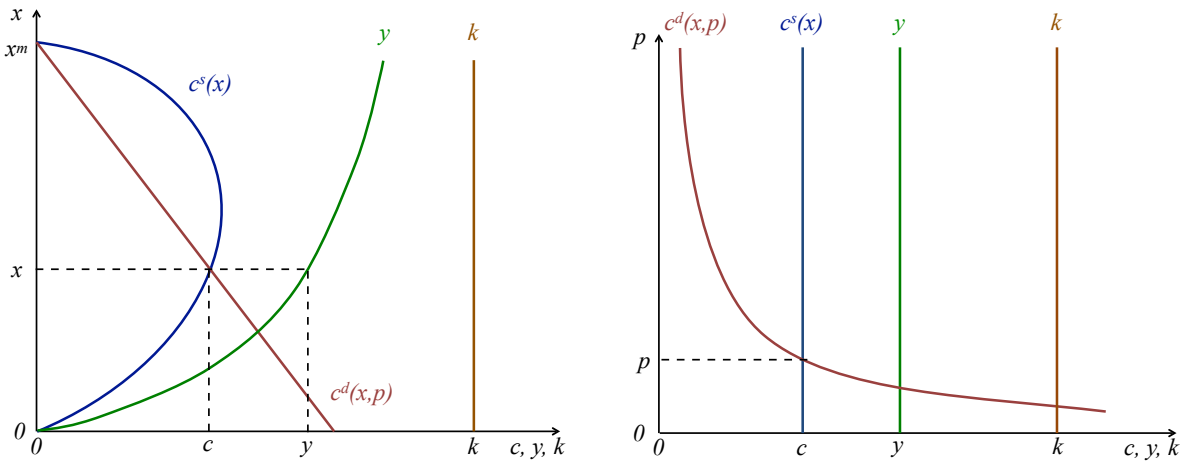
$$c^s(x) = c^d(x, p).$$

Once equilibrium tightness is determined, we infer consumption, output, and idle capacity using the relations summarized in Figure 1.

Since the equilibrium has two variables but one equation, there is one more variable than equilibrium equation, and infinitely many combinations of price and tightness are consistent with the equilibrium definition.[15] To select an equilibrium, we will consider several price mechanisms.

Figure 3 represents aggregate demand and supply, and the equilibrium. Figure 3(a) depicts them in a (c,x) plane. The aggregate demand curve slopes downward. The aggregate supply curve slopes upward for $x \leq x^*$ and downward for $x \geq x^*$. The equilibrium tightness is at the intersection

[15]This indeterminacy is well known. See for instance Howitt and McAfee [1987], Hall [2005], and Farmer [2008]. It arises because each seller-buyer pair decides the price in a situation of bilateral monopoly, whose solution is indeterminate. The situation of bilateral monopoly arises because the pairing of a buyer and a seller generates a positive surplus. The indeterminacy of the solution to the bilateral monopoly problem has been known for a very long time [for example, Edgeworth, 1881].



(a) Representation in a (c,x) plane

(b) Representation in a (c,p) plane

Figure 3: Aggregate Demand, Aggregate Supply, and Equilibrium of the Basic Model of Section 2

of aggregate demand and supply with positive consumption.[16] Figure 3(b) depicts them in a (c,p) plane. Aggregate supply does not depend on the price so the aggregate supply curve is vertical. The aggregate demand curve slopes downward. The equilibrium price is at the intersection of aggregate supply and demand.

2.4 Discussion of the Equilibrium Concept

This section discusses the equilibrium concept presented in Definition 3 by analogy with the Walrasian equilibrium concept. This section is more abstract than the rest of the paper, so it can be skipped on a first reading.

In analogy to Walrasian theory we make the institutional assumption that a price and a tightness are posted on the product market, and we make the behavioral assumption that buyers and sellers take this price and tightness as given. Tightness is the ratio of aggregate number of visits to aggregate capacity. Buyers and sellers take it as given as they are small relative to the size of the market. The issue is more complicated for the price since buyer and seller could bargain the transaction price once they are matched. However, the actual transaction price has no influence on sellers' and buyers' decisions because they are made before the match is realized; what matters is

[16] There is another equilibrium at the other intersection of aggregate demand and supply. In that equilibrium, $x = x^m$ and $c = c^s(x^m) = 0$. We do not focus on that equilibrium because it has zero consumption.

the price at which buyers and sellers expect to trade. Since matching is random, a buyer does not know with which one of the many potential sellers she will trade; we therefore assume that buyers and sellers takes the expected transaction price as given.

To clarify the discussion of the equilibrium concept, we do not use a representative buyer and seller but index buyers by $i \in [0,1]$ and sellers by $j \in [0,1]$. We also assume that sellers can choose their capacity $k(j)$. While k is exogenous in Section 2, it will be endogenous once we introduce firms in Section 3. Definition 3 can be rewritten as follows:

DEFINITION 4. *An* equilibrium *is a price p, market tightness x, a collection of visits $\{v(i),\ i \in [0,1]\}$, and a collection of capacities $\{k(j),\ j \in [0,1]\}$ such that*

(1) Taking x and p as given, buyer $i \in [0,1]$ chooses the number of visits $v(i)$ to maximize her utility subject to her budget constraint and to the constraint imposed by matching frictions: $c(i) = v(i) \cdot q(x)/(1+\tau(x))$, where $c(i)$ is consumption of buyer i.

(2) Taking x and p as given, seller $j \in [0,1]$ chooses the capacity $k(j)$ to maximize her utility subject to the constraint imposed by matching frictions: $y(j) = k(j) \cdot f(x)$, where $y(j)$ is output sold by seller j.

(3) Posted tightness corresponds to actual tightness: $x = \int_0^1 v(i)di / \int_0^1 k(j)dj$

We now discuss how each condition in this definition maps into a condition in of the definition the Walrasian equilibrium.[17]

As in a Walrasian equilibrium, Conditions (1) and (2) imposes that buyers and sellers behave optimally given the quoted price and tightness. A key difference between the two equilibrium concepts is that in a Walrasian equilibrium, buyers and sellers decide the quantity that they desire to trade, whereas in our equilibrium, buyers and sellers decide the number of visits and the number of produced goods for sale, and these lead to a trade only with a certain probability. Buyers decide how many sellers to visit, knowing that each visit leads to a purchase with probability $q(x)$. Sellers decide how much production to offer for sale, knowing that each unit is sold with probability $f(x)$.

In a Walrasian equilibrium, the market clears: at the quoted price, the quantity that buyers desire to buy equals the quantity that sellers desire to sell. This condition can be reformulated as

[17] For a standard definition of the Walrasian equilibrium, see Mas-Colell, Whinston and Green [1995, Chapter 10].

a consistency requirement. Sellers and buyers make their decisions with the expectation that they will be able buy or sell any quantity at the equilibrium price. In other words, they expect that the probability to buy or sell an item is one. For the equilibrium to be consistent with the expectations of sellers and buyers, the quantity that buyers desire to buy must be equal to the quantity that sellers desire to sell such that anybody desiring to trade at the quoted price is able to trade in equilibrium. This condition can only be fulfilled if the market clears.

Condition (3) is the equivalent to this consistency requirement in the presence of matching frictions. Once buyers have chosen $\{v(i),\ i \in [0,1]\}$ and sellers have chosen $\{k(j),\ j \in [0,1]\}$, the number of trades is

$$\left[\left(\int v(i)di\right)^{-\gamma} + \left(\int k(j)dj\right)^{-\gamma}\right]^{-\frac{1}{\gamma}} = \int k(j)dj \cdot f\left(\frac{\int v(i)di}{\int k(j)dj}\right) = \int v(i)di \cdot q\left(\frac{\int v(i)di}{\int k(j)dj}\right).$$

These equalities imply that the selling probability faced by sellers is $f\left(\int v(i)di / \int k(j)dj\right)$ and the buying probability faced by buyers is $q\left(\int v(i)di / \int k(j)dj\right)$. Both probabilities do not have to be equal to the probabilities on which sellers and buyers based their calculations, $f(x)$ and $q(x)$. In equilibrium, we impose the consistency requirement that these probabilities match, or equivalently, that the posted tightness equals the actual tightness: $x = \int v(i)di / \int y(j)dj$.

Let us explain why the Definitions 3 and 4 are equivalent. Given the definition of aggregate demand, Condition (1) imposes that $v(x,p) = (1+\tau(x)) \cdot c^d(x,p)/q(x)$. Here there is no active decision from the seller: each seller provides an exogenous amount k. Therefore, Condition (2) imposes that $k(x,p) = k$. Condition (3) imposes that $x = v(x,p)/k(x,p) = (1+\tau(x)) \cdot c^d(x,p)/(k \cdot q(x))$. We can rewrite this condition as

$$c^d(x,p) = \frac{x \cdot q(x)}{1+\tau(x)} \cdot k = \frac{f(x)}{1+\tau(x)} \cdot k = c^s(x).$$

This is the condition that aggregate supply equals aggregate demand in Definition 3.

Under these equilibrium conditions, the budget constraints of all households are satisfied, and sales equal purchases through the matching process. Thus, following Walras' law, the market for nonproduced good necessarily clears.

2.5 Welfare

Here we study the welfare properties of the equilibrium. We first define the efficient allocation:

DEFINITION 5. *An* efficient allocation *is a consumption and a product market tightness* (c,x) *that maximize welfare subject to the matching frictions,* $c \leq (f(x) - \rho \cdot x) \cdot k$.

Welfare is given by

$$\left(\frac{\chi}{1+\chi} \cdot c^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+\chi} \cdot \mu^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{8}$$

Hence, maximizing welfare is equivalent to maximizing consumption.

PROPOSITION 1. *There exists a unique efficient allocation* (c^*,x^*). *The allocation satisfies* $c^* = [f(x^*) - \rho \cdot x^*] \cdot k$ *and* $f'(x^*) = \rho$. *Equivalently,* x^* *satisfies*

$$\tau(x^*) = \frac{1-\eta(x^*)}{\eta(x^*)}, \tag{9}$$

where $1-\eta(x)$ *is the elasticity of* $f(x)$. *In particular,* $x^* \in (0,x^m)$.

The proof of the proposition is in Appendix A, along with the proofs of all the other propositions in the text. The efficient allocation is the furthest point to the right on the aggregate supply curve, as showed in Figure 4. At this point, the aggregate supply is maximized. Equation (9) is useful to assess empirically the efficiency of a market with matching frictions, as explained in Landais, Michaillat and Saez [2010].

The equilibrium of Definition 3 may not be efficient because implicitly, the price is determined through bilateral bargaining.[18] When the equilibrium is not efficient, it can be either tight or slack.

DEFINITION 6. *An equilibrium is* efficient *if* $x = x^*$, slack *if* $x < x^*$, *and* tight *if* $x > x^*$.

Figure 4 illustrates these three regimes. In a slack equilibrium, aggregate demand is too low and tightness is below its efficient level; consumption and output are below their efficient level. In a tight equilibrium, aggregate demand is too high and tightness is above its efficient level; consumption is again below its efficient level but output is above its efficient level. Note that

[18] This statement is true in any matching model. See Hosios [1990] and Pissarides [2000, Chapter 8].

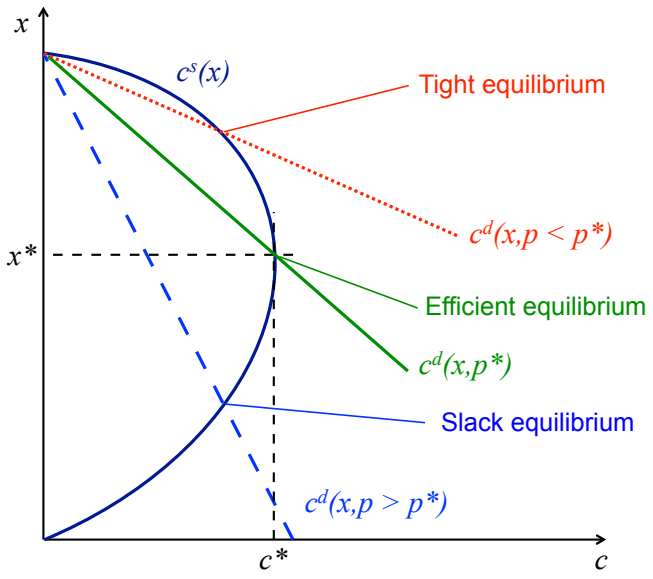


Figure 4: The Three Regimes of the Basic Model of Section 2

Notes: The figure compares the equilibria obtained for different equilibrium prices. The price p^* is given by (11).

higher consumption always implies higher welfare, which is not the case of higher output. Given that the aggregate demand is increasing in price, it is clear that the price is above its efficient level when the equilibrium is slack, and below its efficient level when the equilibrium is tight.

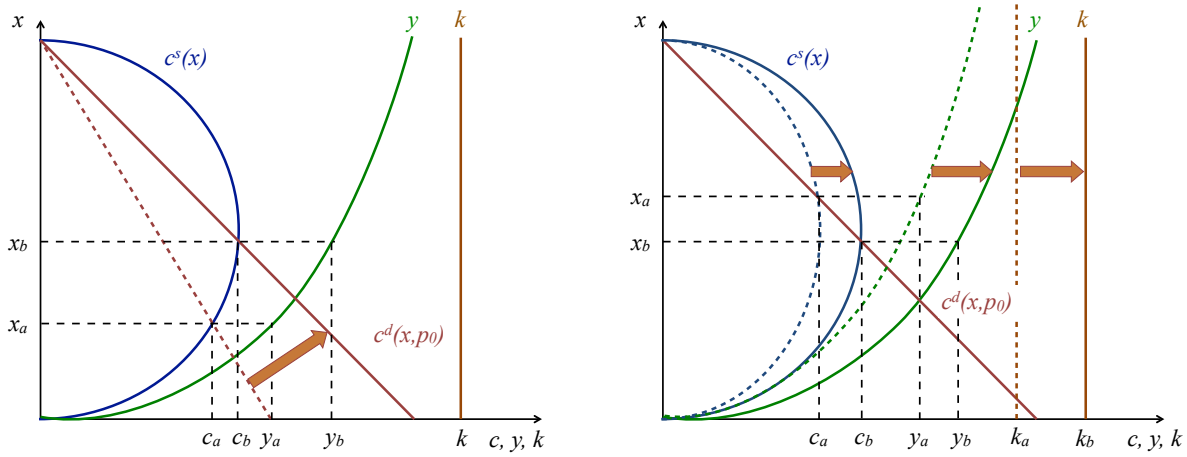
2.6 Fixprice Equilibrium

We now consider different equilibria in which we assume different price mechanisms. We first study a simple equilibrium in which the price is a parameter.[19] In this equilibrium, only tightness equilibrates the market.

DEFINITION 7. *A* fixprice equilibrium *parameterized by* $p_0 > 0$ *consists of a product market tightness and a price* (x, p) *such that aggregate supply equals aggregate demand and the price is given by the parameter* p_0*:* $c^s(x) = c^d(x, p)$ *and* $p = p_0$.

PROPOSITION 2. *For any* $p_0 > 0$*, there exits a unique fixprice equilibrium parameterized by* p_0 *with positive consumption.*

[19] In matching models of the labor market, several researchers have assumed that the wage is a parameter or a function of the parameters. See for instance Hall [2005], Blanchard and Galí [2010], and Michaillat [2012, 2014].



(a) Increase in aggregate demand

(b) Increase in aggregate supply

Figure 5: Shocks in the Fixprice Equilibrium of the Basic Model of Section 2

We now study the comparative static effects of aggregate demand shocks and aggregate supply shocks. The comparative statics are summarized in Table 1. The price is a parameter so it is fixed in the comparative statics.

We parameterize an increase in aggregate demand by an increase in the taste for produced good, χ, or in the endowment of nonproduced good, μ. By manipulating the equilibrium condition $c^s(x) = c^d(x, p_0)$, we find that the equilibrium tightness is the unique solution to

$$(1+\tau(x))^{\varepsilon-1} \cdot f(x) = \frac{\mu}{k} \cdot \left(\frac{\chi}{p_0}\right)^{\varepsilon}. \tag{10}$$

Since τ and f are strictly increasing functions of x and $\varepsilon > 1$, equation (10) implicitly defines x as an increasing function of μ and χ. Therefore, tightness increases after an increase in aggregate demand. The intuition is that households want to consume more produced good so workers sell a larger fraction of a fixed amount of production. Since tightness increases, the fraction of time when workers are idle, $1-f(x)$, decreases but output, $y = f(x) \cdot k$, increases. The impact on consumption, $c = c^s(x)$, depends on the regime: in the slack regime, $dc^s/dx > 0$ so consumption increases; in the efficient regime, $dc^s/dx = 0$ so consumption does not change; and in the tight regime, $dc^s/dx < 0$ so consumption falls. In the tight regime, a higher tightness reduces the output devoted to consumption even though it increases total output because it increases sharply the share of output dissipated in matching frictions. Figure 5(a) depicts an increase in aggregate demand. The

aggregate demand curve rotates outward; therefore, product market tightness and output increase. The finding that aggregate demand matters with rigid prices echoes the findings of a vast body of work in macroeconomics, including the seminal contributions of Barro and Grossman [1971] and Blanchard and Kiyotaki [1987].

We parameterize an increase in aggregate supply by an increase in capacity, k. Equation (10) implicitly defines x as a decreasing function of k. Therefore, tightness decreases after an increase in aggregate supply. The intuition is that workers offer more production for sale but households do not desire to consume more at a given price, so workers sell a smaller fraction of a larger amount of production. Since tightness decreases, the fraction of time when workers are idle, $1 - f(x)$, increases, and consumption, $c = c^d(x, p)$, increases. The intuition is that the effective price of produced good, $(1 + \tau(x)) \cdot p$, falls, which stimulates households to increase consumption. Since k increases but x falls, the impact on output, $y = f(x) \cdot k$, is not obvious. But in equilibrium, output satisfies $y = (1 + \tau(x)) \cdot c^d(x, p) = (1 + \tau(x))^{1-\varepsilon} \cdot \mu \cdot \chi^\varepsilon / p^\varepsilon$. As x falls, $(1 + \tau(x))^{\varepsilon-1}$ falls since $\varepsilon > 1$, and hence y increases. Figure 5(b) depicts an increase in aggregate supply. The shock leads the aggregate supply curve to expand, raising consumption and output but reducing product market tightness.

Aggregate supply and aggregate demand shocks have different macroeconomic effects. Product market tightness and output are positively correlated under aggregate demand shocks but negatively correlated under aggregate supply shocks. We will exploit this property in the empirical analysis in Section 5.

2.7 Efficient Equilibrium

We now study an equilibrium in which the price always ensures efficiency.

DEFINITION 8. *An* efficient equilibrium *consists of a product market tightness and a price* (x, p) *such that aggregate supply equals aggregate demand and tightness is efficient:* $c^s(x) = c^d(x, p)$ *and* $x = x^*$.

PROPOSITION 3. *There exits a unique efficient equilibrium. The efficient price is*

$$p^* = (1 + \tau(x^*))^{-\frac{\varepsilon-1}{\varepsilon}} \cdot f(x^*)^{-\frac{1}{\varepsilon}} \cdot \chi \cdot \left(\frac{\mu}{k}\right)^{\frac{1}{\varepsilon}}. \tag{11}$$

Table 1: Comparative Statics in the Basic Model of Section 2

	Effect on:			
Increase in:	Output y	Product market tightness x	Idle time $1-f(x)$	Consumption c
		A. Fixprice equilibrium		
Aggregate demand	+	+	−	+ (slack) 0 (efficient) − (tight)
Aggregate supply	+	−	+	+
		B. Efficient equilibrium		
Aggregate demand	0	0	0	0
Aggregate supply	+	0	0	+

Notes: An increase in aggregate demand is an increase in endowment of nonproduced good, μ, or an increase in the taste for produced good, χ. An increase in aggregate supply is an increase in capacity, k.

The price p^* is such that the aggregate demand curve intersects the aggregate supply curve at the efficient allocation. This price necessarily exists because by increasing the price from 0 to $+\infty$, the aggregate demand curve rotates around the point $(0, x^m)$ from a horizontal position to a vertical position. The efficient price ensures that the aggregate demand curve is always in the position depicted in Figure 4. This price corresponds to the price characterized by Hosios [1990] in his seminal analysis of the efficiency properties of matching models.

With an efficient price, equilibrium product market tightness is set at $x = x^*$, where x^* satisfies $f'(x^*) = \rho$. Since the level of product market tightness is solely determined by the matching function and the matching cost, product market tightness responds neither to aggregate demand nor aggregate supply shocks. Output and consumption are given by $y = f(x) \cdot k$ and $c = [f(x) - \rho \cdot x] \cdot k$; thus, output and consumption do not respond to aggregate demand shocks (both x and k remain constant) but they do respond to aggregate supply shocks (x remains constant but k moves). These results echo the findings of Blanchard and Galí [2010] and Shimer [2010, Chapter 2] that labor market tightness does not respond to shocks when the wage is efficient in matching models of the labor market.

A large literature has argued that through the directed search mechanism of Moen [1997],

market forces would always maintain price and tightness at their efficient levels.[20] Under the directed search mechanism, prices could not be rigid because market forces would immediately push sellers who do not adjust their price out of business. While this mechanism could operate in the long-run, it may not operate perfectly at business-cycle frequency as creating new markets selling the same product at different prices can take time. At a minimum, the directed search mechanism does not operate at hourly or daily frequency: queues at restaurants systematically vary depending on the time of the day or the day of the week, which indicates that prices do not adjust sufficiently to absorb variations in demand. Section 5 will provide an empirical test of price rigidity.

The Walrasian equilibrium can be seen as a special case of the efficient equilibrium when the matching cost, ρ, goes to 0. First, when $\rho = 0$, $\tau(x) = 0$ so the effective price of consumption is the market price as in the Walrasian case; there is no matching wedge. Second, when $\rho = 0$, the aggregate supply curve becomes the same as the output curve: $c^s(x) = f(x) \cdot k$. The efficient tightness maximizes consumption and therefore $f(x)$; accordingly, $x^* = \infty$ and $f(x^*) = 1$. Hence, households sell all their production as in the Walrasian case; there is no idle time.

We can therefore replicate a Walrasian economy with a matching framework by assuming that $\rho = 0$ and that the price is efficient. However, households do not always sell their entire capacity when $\rho = 0$. If the equilibrium price is higher than the efficient price, some idle time prevails even if $\rho = 0$.[21] Figure 3(a) can be used to illustrate what happens when $\rho = 0$. The aggregate supply curve takes the shape of the output curve and the aggregate demand curve becomes vertical with $c^d = \chi^\varepsilon \cdot \mu \cdot p^{-\varepsilon}$. Hence, if $p > p^* = \chi \cdot (\mu/k)^{1/\varepsilon}$, then $c^d < k$, not all production is sold, and workers are idle part of the time.

[20] The mechanism operates as follows. Starting from an equilibrium (p_a, x_a), a small subset of sellers can deviate and offer a different price, p_b. Buyers will flee or flock to the new sellers until they are indifferent between old and new markets, which happens when $p_a \cdot (1+\tau(x_a)) = p_b \cdot (1+\tau(x_b))$. Deviating sellers then obtain a revenue $p_b \cdot f(x_b)$ instead of $p_a \cdot f(x_a)$. Hence, sellers' optimal choice is to select p_b to maximize $p_b \cdot f(x_b)$ subject to $p_a \cdot (1+\tau(x_a)) = p_b \cdot (1+\tau(x_{ab}))$. This is equivalent to selecting x_b to maximize $f(x_b)/[1+\tau(x_b)] = (f(x_b) - \rho x_b)$—that is, to selecting the efficient tightness, x^*.

[21] Michaillat [2012] obtained the same result in the context of a labor market matching model.

2.8 Other Equilibria

We have considered two polar cases: fixed price and efficient price. However, many other price mechanisms are possible. We study two of them here: Nash bargaining and partially rigid price.

Bargaining Equilibrium. The most common mechanism to set prices and select an equilibrium in the matching literature is Nash bargaining.[22] We show that even though the economy is not necessarily efficient, the comparative statics are exactly the same under Nash bargaining as under efficient pricing.

Following the literature on Nash bargaining, we assume that households have a linear utility function $(\chi \cdot c + m)/(1+\chi)$, which is the special case of our utility function when $\varepsilon \to +\infty$. The optimal consumption choice of households, given by (6), yields the following aggregate demand equation: $(1+\tau(x)) \cdot p = \chi$. The aggregate demand is perfectly elastic with respect to x; in Figure 3(a), the aggregate demand would be represented by a horizontal curve.

To determine equilibrium tightness, we need the equilibrium price. The price is the generalized Nash solution to the bargaining problem between a buyer and a seller with bargaining power $\beta \in (0,1)$. After a match has been made, the surplus to the household of buying one unit of produced good is $\mathscr{C}(p) = [\chi/(1+\chi)] - [p/(1+\chi)]$, and the surplus to the household of selling one unit of produced good is $\mathscr{F}(p) = p/(1+\chi)$. The Nash solution maximizes $\mathscr{C}(p)^{1-\beta} \cdot \mathscr{F}(p)^{\beta}$, so that $\mathscr{F}(p) = \beta \cdot [\mathscr{F}(p) + \mathscr{C}(p)] = \beta \cdot \chi/(1+\chi)$ and the price is $p = \beta \cdot \chi$.

Combining the aggregate demand condition and the bargained price, we obtain equilibrium tightness:

$$\beta \cdot (1+\tau(x)) = 1. \tag{12}$$

Comparing this equilibrium condition with the efficiency condition (9), we infer that the equilibrium is efficient if and only if $\beta = \eta(x)$, which is exactly the Hosios [1990] condition. Hence, the bargaining equilibrium is efficient only for a specific bargaining power, and not generally.

The comparative statics are obvious. Tightness responds neither to aggregate demand shocks (shocks to χ) nor to aggregate supply shocks (shocks to k). Since output is $y = f(x) \cdot k$ and con-

[22] Nash bargaining was first used in the seminal work of Diamond [1982], Mortensen [1982], and Pissarides [1985].

sumption is $(f(x) - \rho \cdot x) \cdot k$, output and consumption do not respond to aggregate demand shocks but respond positively to aggregate supply shocks. To conclude, we obtain exactly the comparative statics of the efficient equilibrium. Our comparative static results are consistent with the results of Blanchard and Galí [2010] and Michaillat [2012], who found that with Nash bargaining in matching models of the labor market, labor market tightness does not respond at all to labor demand shocks in the form of technology shocks.

Equilibrium with Partially Rigid Price. We generalize the fixprice equilibrium to an equilibrium in which the price responds partially to underlying shocks. We show that the comparative statics remain exactly the same as in the fixprice equilibrium.

The efficient price, given by (11), is a benchmark that defines price flexibility. We define a partially rigid price as a departure from this benchmark. We assume that the price is a function of the parameters given by

$$p = p_0 \cdot \chi^{\xi} \cdot \left(\frac{\mu}{k}\right)^{\frac{\xi}{\varepsilon}}, \tag{13}$$

where $p_0 > 0$ and $\xi \in [0,1)$ parameterizes price rigidity. If $\xi = 0$, the price is completely rigid: it is a parameter as in the fixprice equilibrium. If $\xi = 1$, the price is fully flexible: it is proportional to $\chi \cdot (\mu/k)^{1/\varepsilon}$, like the efficient price.

In equilibrium, tightness equalizes aggregate demand and supply, with the price given by (13). As in the fixprice case, there exists a unique equilibrium with positive consumption. Combining $c^s(x) = c^d(x,p)$ with (13), we find that equilibrium tightness satisfies

$$(1+\tau(x))^{\varepsilon-1} \cdot f(x) = \left(\frac{\chi^{\varepsilon} \cdot \mu}{k}\right)^{1-\xi} \cdot \frac{1}{p_0^{\varepsilon}}.$$

Since $\xi < 1$, all the comparative statics of product market tightness are the same as those obtained in the fixprice equilibrium with (10). That is, the comparative statics of the fixprice equilibrium remain valid if the price is partially rigid instead of being completely rigid. These results echo the findings of Blanchard and Galí [2010] and Michaillat [2012] that in matching models of the labor market, the qualitative effects of technology shocks on labor market tightness are the same whether the wage is fixed or partially rigid with respect to technology.

The analysis of the equilibrium with partially rigid prices implies that the comparative statics obtained in the fixprice equilibrium are very robust: they only break down in the knife-edge case in which the price is fully flexible and responds one-for-one to the underlying shocks; when the price is partially but not fully flexible, the comparative statics of the fixprice equilibrium obtain.

3 Model of Aggregate Demand, Idle Time, and Unemployment

This section develops the main model of the paper. The model retains the architecture of the Barro and Grossman [1971] model but replaces the disequilibrium framework on the labor and product markets by a matching framework. The model allows us to study the effect of a broad set of shocks—labor supply shocks, mismatch shocks, technology shocks, or aggregate demand shocks—on the labor market.

3.1 Assumptions

The economy has a measure 1 of identical households and a measure 1 of identical firms, owned by the households. Household members pool their income before jointly deciding consumption. The product market is the same as in Section 2. The only difference is that the capacity of firms is not exogenous but determined endogenously from the production decision of firms. The labor market is isomorphic to the product market. In this section we only describe the new parts of the model: the labor market and firms.

The Labor Market. The number of household members in the labor force is $h \in (0,1)$. The $1-h$ other household members are out of the labor force. There are matching frictions on the labor market. All labor force participants are initially unemployed and search for a job. Each firm posts $\hat{v}$ vacancies to hire workers. The number l of workers who find a job is given by the following matching function: $l = \left(h^{-\hat{\gamma}} + \hat{v}^{-\hat{\gamma}}\right)^{-1/\hat{\gamma}}$. Labor market tightness is defined as the ratio of vacancy to initial unemployment: $\theta \equiv \hat{v}/h$. Labor market tightness determines the probabilities that a jobseeker finds a job and a vacancy is filled. Jobseekers find a job with probability $\hat{f}(\theta) = l/h = \left(1+\theta^{-\hat{\gamma}}\right)^{-1/\hat{\gamma}}$, and a vacancy is filled with probability $\hat{q}(\theta) = l/\hat{v} = \left(1+\theta^{\hat{\gamma}}\right)^{-1/\hat{\gamma}}$. The functions $\hat{f}$ and $\hat{q}$ have the same properties as the functions f and q. Hence, it is easier to find a job

but harder to fill a vacancy when the labor market tightness is higher. We assume away randomness at the firm and household levels: each firm hires $\hat{v} \cdot \hat{q}(\theta)$ workers for sure, and $\hat{f}(\theta) \cdot h$ household members find a job for sure.

They are two types of employees: n producers and $l - n$ recruiters. We assume that posting a vacancy requires a fraction $\hat{\rho} > 0$ of a recruiter's time. Thus, the number of recruiters is $l - n = \hat{\rho} \cdot \hat{v} = \hat{\rho} \cdot l / \hat{q}(\theta)$. The number of producers is therefore related to the total number of workers by $l = (1 + \hat{\tau}(\theta)) \cdot n$, where $\hat{\tau}(\theta) \equiv \hat{\rho} / (\hat{q}(\theta) - \hat{\rho})$ measures the number of recruiters per producer. The function $\hat{\tau}$ as the same properties as the function τ.

Figure 6(a) illustrates the matching frictions on the labor market. The labor market is isomorphic to the product market because the matching process is similar on these two markets. Employment is $l = \hat{f}(\theta) \cdot h$ so the employment curve is increasing and concave with tightness. The number of producers is $n = \hat{f}(\theta) \cdot h / (1 + \hat{\tau}(\theta)) = \left(\hat{f}(\theta) - \hat{\rho} \cdot \theta\right) \cdot h$; so this number increases with θ for $\theta \leq \theta^*$ and then decreases for $\theta \geq \theta^*$. There are $l - n = \hat{\rho} \cdot \hat{v} = \hat{\rho} \cdot \theta \cdot h$ recruiters. The gap between labor supply curve and employment curve gives the number of recruiters. There are $h - l = (1 - \hat{f}(\theta)) \cdot h$ unemployed workers. The gap between employment curve and labor force gives the number of unemployed workers.

The producer curve corresponds to the labor supply. We define the labor supply as the number of producers employed for a given tightness, taking into account that households supply a fixed quantity h of labor:

DEFINITION 9. *The* labor supply *is a function of labor market tightness defined by*

$$n^s(\theta) = \frac{\hat{f}(\theta)}{1 + \hat{\tau}(\theta)} \cdot h = \left(\hat{f}(\theta) - \hat{\rho} \cdot \theta\right) \cdot h$$

for all $\theta \in [0, \theta^m]$*, where* $\theta^m > 0$ *satisfies* $\hat{\rho} = \hat{q}(\theta^m)$*. The function* n^s *is strictly increasing on* $[0, \theta^*]$ *and strictly decreasing on* $[\theta^*, \theta^m]$*, where* $\theta^* > 0$ *satisfies* $\hat{f}'(\theta^*) = \hat{\rho}$*. We also have* $n^s(0) = 0$*, and* $n^s(\theta^m) = 0$*.*

Firms. The representative firm hires l workers. Some of the workers are engaged in production while others are engaged in recruiting. More precisely, $n < l$ producers produce a quantity k of output according to the production function $k = a \cdot n^\alpha$. The parameter $a > 0$ measures the

technology of the firm and the parameter $\alpha \in (0,1)$ captures decreasing marginal returns to labor. Because of the product market frictions, the firm only sells a fraction $f(x)$ of its capacity k.

The firm pays its l workers a real wage w; the nominal wage is $p \cdot w$. The real wage bill of the firm is $w \cdot l = (1+\hat{\tau}(\theta)) \cdot w \cdot n$. From this perspective, matching frictions in the labor market impose a wedge $\hat{\tau}(\theta)$ on the wage of producers.

Given θ, x, p, and w, the firm chooses n to maximize profits

$$\pi = p \cdot f(x) \cdot a \cdot n^{\alpha} - (1+\hat{\tau}(\theta)) \cdot p \cdot w \cdot n.$$

Hence, the optimal number of producers n satisfies:

$$f(x) \cdot \alpha \cdot a \cdot n^{\alpha-1} = (1+\hat{\tau}(\theta)) \cdot w. \tag{14}$$

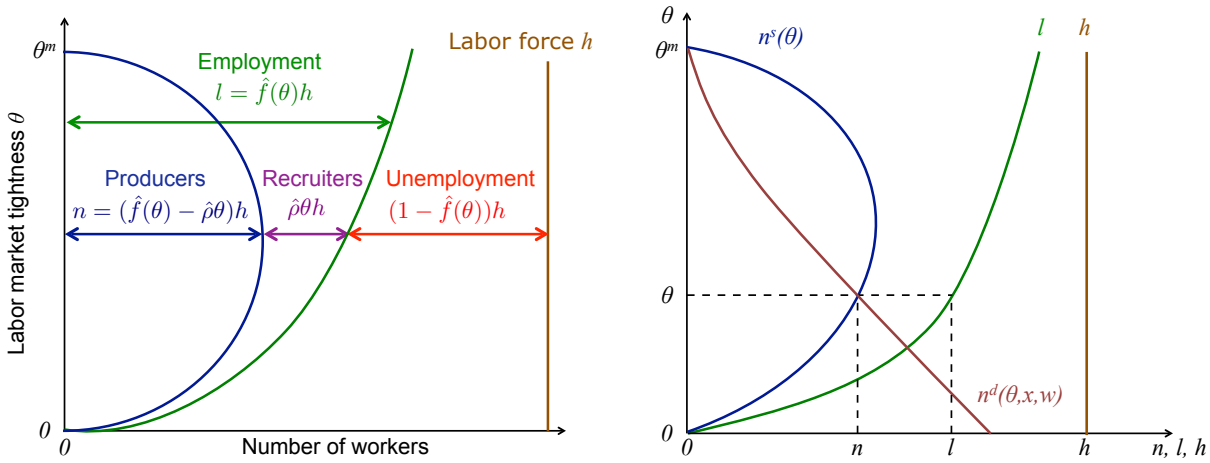
At the optimum, the real marginal revenue of one producer equals the real marginal cost of one producer. The real marginal revenue is the marginal product of labor, $\alpha \cdot a \cdot n^{\alpha-1}$, times the selling probability, $f(x)$. The real marginal cost is the real wage, w, plus the marginal recruiting cost, $\hat{\tau}(\theta) \cdot w$. Using (14), we define the labor demand as the optimal number of producers to hire given the product market tightness, labor market tightness, and real wage:

DEFINITION 10. *The* labor demand *is a function of labor market tightness, product market tightness, and real wage defined by*

$$n^d(\theta,x,w) = \left[\frac{f(x) \cdot \alpha \cdot a}{(1+\hat{\tau}(\theta)) \cdot w}\right]^{\frac{1}{1-\alpha}}$$

for all $(\theta,x,w) \in [0,\theta^m] \times (0,+\infty) \times (0,+\infty)$*, where* $\theta^m > 0$ *satisfies* $\hat{\rho} = \hat{q}(\theta^m)$*. The function* n^d *is strictly decreasing in* θ*, strictly increasing in* x*, and strictly decreasing in* w*. We also have* $n^d(0,x,w) = [f(x) \cdot \alpha \cdot a \cdot (1-\hat{\rho})/w]^{1/(1-\alpha)}$ *and* $n^d(\theta^m,x,w) = 0$*.*

The labor demand is strictly decreasing in w and θ because when either of them increases, the effective wage of a producer, $(1+\hat{\tau}(\theta)) \cdot w$, increases and firms reduce hiring of producers. The labor demand is strictly increasing in x because when x increases, the probability $f(x)$ to sell output increases and firms increase hiring of producers. The labor demand is depicted on Figure 6(b) in



(a) The matching frictions on the labor market

(b) Labor demand and supply in a (n, θ) plane

Figure 6: The Labor Market in the Model of Section 3

the (n, θ) plane, together with the labor supply. The labor demand slopes downward.

3.2 Definition of the Equilibrium

In this model the aggregate demand is given by (7) and the aggregate supply is given by

$$c^s(x,\theta) = (f(x) - \rho \cdot x) \cdot a \cdot \left(\frac{\hat{f}(\theta)}{1+\hat{\tau}(\theta)}\right)^{\alpha} \cdot h^{\alpha}.$$

This definition is the same as (3), except that the capacity is now endogenous and given by $a \cdot n^{\alpha}$, where n is expressed as a function of θ using the labor supply. We can now define the equilibrium:

DEFINITION 11. *An* equilibrium *consists of a product market tightness, a price, a labor market tightness, and a real wage* (x, p, θ, w) *such that aggregate supply is equal to aggregate demand and labor supply is equal to labor demand:*

$$\begin{cases} c^s(x,\theta) &= c^d(x,p) \\ n^s(\theta) &= n^d(\theta,x,w) \end{cases}$$

Once x and θ are determined, we infer equilibrium consumption, output, idle time, number of producers, employment, and unemployment using the relations summarized in Figures 1 and 6(a).

Since the equilibrium is composed of four variables that satisfy two equations, infinitely many combinations of (x, p, θ, w) are consistent with the definition. To select an equilibrium, we will consider several price and wage mechanisms.

3.3 The Factors of Unemployment

The general disequilibrium theory of Barro and Grossman [1971] advanced macroeconomics by proposing a model in which unemployment can be caused either by excessive real wages (*classical* unemployment) or by deficient aggregate demand (*Keynesian* unemployment). Our model extends the general disequilibrium theory by proposing a model that captures the existence of the three canonical types of unemployment: classical and Keynesian unemployment, together with *frictional* unemployment. Compared to a standard matching model, in which unemployment can be frictional and classical, our model adds a layer of Keynesian unemployment because it accounts for the difficulty of firms to sell production, which materializes itself as idle time for employed workers.

Equilibrium employment is related to the labor demand by $l = (1 + \hat{\tau}(\theta)) \cdot n^d(\theta, x, w)$, so that

$$l = \left(\frac{f(x) \cdot \alpha \cdot a}{w} \right)^{\frac{1}{1-\alpha}} \cdot \left(\frac{1}{1 + \hat{\tau}(\theta)} \right)^{\frac{\alpha}{1-\alpha}}, \tag{15}$$

where θ, x, and w are equilibrium labor market tightness, product market tightness, and real wage. The model captures frictional unemployment through $\hat{\tau}(\theta) > 0$. Employment is lowered because firms incur a cost to fill vacancies. The model captures Keynesian unemployment through $f(x) < 1$. Employment is lowered because firms may not be able to sell all their production, or equivalently, to utilize their labor fully. The model also captures classical unemployment through $w > \alpha \cdot a \cdot h^{\alpha-1}$. Employment is lowered because firms may pay a real wage that is above the marginal product of labor of the last worker in the labor force.

The mechanism giving rise to Keynesian unemployment in our model is different from the one in the Barro-Grossman model. In the Barro-Grossman model, if the price is above market-clearing level, the quantity of output is determined by the aggregate demand and the price level, and firms hire just the number of workers needed to produce this output. The same mechanism holds in a variety of models with rigid prices, in which quantities are determined by demand. In contrast, in our model, aggregate demand determines idle time which in turn affects labor demand and hence

unemployment.

The mechanism in our model has the advantage of being consistent with the empirical findings of the literature on endogenous productivity. This literature, initiated by Hall [1988], Evans [1992], and Burnside, Eichenbaum and Rebelo [1993], has established empirically that labor productivity responds positively to exogenous aggregate demand shocks. In the Barro-Grossman model, $y = a \cdot n^{\alpha}$, hence labor productivity, measured as y/n^{α}, is constant in response to aggregate demand shocks. In our model, $y = f(x) \cdot a \cdot n^{\alpha}$ and hence $y/n^{\alpha} = a \cdot f(x)$. Therefore, labor productivity moves with product market tightness. Since an increase in aggregate demand raises product market tightness (see Table 2 below), it also raises labor productivity, consistent with empirical evidence.

3.4 Welfare

We begin by defining and describing the efficient allocation:

DEFINITION 12. *An* efficient allocation *is a consumption, product market tightness, number of producers, and labor market tightness* (c,x,n,θ) *that maximize welfare subject to the matching frictions on the product and labor markets:* $c \leq (f(x) - \rho \cdot x) \cdot a \cdot n^{\alpha}$ *and* $n \leq \left(\hat{f}(\theta) - \hat{\rho} \cdot \theta\right) \cdot h$.

PROPOSITION 4. *There exists a unique efficient allocation* (c^*,x^*,n^*,θ^*). *The allocation satisfies* $c^* = [f(x^*) - \rho \cdot x^*] \cdot a \cdot (n^*)^{\alpha}$, $n^* = \left[\hat{f}(\theta^*) - \hat{\rho} \cdot \theta^*\right] \cdot h$, $f'(x^*) = \rho$, *and* $\hat{f}'(\theta^*) = \hat{\rho}$. *Equivalently,* x^* *and* θ^* *satisfy* $\tau(x^*) = (1 - \eta(x^*))/\eta(x^*)$ *and* $\hat{\tau}(\theta^*) = (1 - \hat{\eta}(\theta^*))/\hat{\eta}(\theta^*)$ *where* $1 - \eta(x)$ *is the elasticity of* $f(x)$ *and* $1 - \hat{\eta}(\theta)$ *is the elasticity of* $\hat{f}(\theta)$.

The equilibrium may not be efficient, in which case it may be in four different regimes:

DEFINITION 13. *The equilibrium is* efficient *if* $\theta = \theta^*$ *and* $x = x^*$, labor-slack and product-slack *if* $\theta < \theta^*$ *and* $x < x^*$, labor-slack and product-tight *if* $\theta < \theta^*$ *and* $x > x^*$, labor-tight and product-slack *if* $\theta > \theta^*$ *and* $x < x^*$, labor-tight and product-tight *if* $\theta > \theta^*$ *and* $x > x^*$.

These four inefficient regimes are reminiscent of the four regimes from the Barro-Grossman model. In both models, the regimes are determined by the price and real wage being inefficiently high or low. It is possible to characterize precisely the four regions of a (w,p) plane corresponding to the four types of inefficient equilibria. In that plane, the region for the labor-slack and product-slack equilibria has high price and high real wage, the region for labor-tight and product-tight

equilibria has low price and low real wage, and so on. But there are important differences: in our model, the equilibrium is determined by the same system of smooth equations in all the types of equilibria (the only difference between the four types of equilibria are the slopes of the aggregate supply and labor supply); but in the general disequilibrium model, each regime is described by a distinct system of equations, depending on whether the supply or the demand binds in each market; our representation of the four regimes is therefore more tractable.

3.5 Fixprice Equilibrium

We extend the definition and analysis of the fixprice equilibrium to the model with product and labor market.

DEFINITION 14. *A* fixprice equilibrium *parameterized by* $p_0 > 0$ *and* $w_0 > 0$ *consists of a product market tightness, a price, a labor market tightness, and a real wage* (x, p, θ, w) *such that supply equals demand on the product and labor markets and price and real wage are given by the parameter* p_0 *and* w_0*:* $c^s(x,\theta) = c^d(x,p)$*,* $n^s(\theta) = n^d(\theta,x,w)$*,* $p = p_0$*, and* $w = w_0$*.*

PROPOSITION 5. *For any* $p_0 > 0$ *and* $w_0 > 0$*, there exists a unique fixprice equilibrium parameterized by* $p_0 > 0$ *and* $w_0 > 0$ *with positive consumption.*

We use comparative statics to describe the response of the equilibrium to four types of shocks: aggregate demand shock, technology shock, labor supply shock, and labor market mismatch shock. We focus on the response of product market tightness, labor market tightness, output, and employment. The response of all the other variable can be derived from the relations in Figures 1 and 6(a). The comparative statics are summarized in Table 2.[23] Here we discuss these comparative statics with the help of the equilibrium diagrams. The formal derivations are relegated to Appendix B.

First, we study aggregate demand shocks. We parameterize an increase in aggregate demand by an increase in taste for produced good, χ, or in endowment of nonproduced good, μ. After an increase in aggregate demand, product and labor market tightness increase. Output and employment increase as well. The comparative static effects of an increase in aggregate demand can be illustrated with the equilibrium diagrams. Since the price is fixed, the shock leads to an upward

[23] With a linear production function ($\alpha = 1$), all the comparative statics would remain the same.

rotation of the aggregate demand curve in Figure 3(a), which raises product market tightness. The probability to sell is higher so idle time falls and hiring a worker becomes more profitable. In Figure 6(b), the labor demand curve rotates outward, which raises labor market tightness. Since labor market tightness rises, the aggregate supply curve shifts outward in Figure 3(a), which tends to lower product market tightness. Overall, the direct effects dominate so product and labor market tightness increase.

Second, we study technology shocks. We parameterize an increase in technology by an increase in the production-function parameter, a. After an increase in technology, labor market tightness increases but product market tightness decreases. Both output and employment increase. In Figure 6(b), an increase in technology leads to an upward rotation of the labor demand curve because with rigid wages, an increase in technology raises the profitability of hiring workers. This rotation leads to higher labor market tightness. In Figure 3(a), an increase in technology leads to an outward shift of the aggregate supply curve, which lowers product market tightness. Since product market tightness falls, the labor demand rotates back inward; and since labor market tightness increases, the aggregate supply shifts further outward. Overall, the direct effects dominate so product market tightness decreases while labor market tightness increases.

Third, we study labor supply shocks. We parameterize an increase in labor supply by an increase in h, which represents an exogenous increase in labor force participation for demographic factors, or an exogenous increase in labor force participation because the taste for leisure and work changed, or a change in job-search effort arising from exogenous policy changes not modeled here, such as a change in the generosity of disability insurance or unemployment insurance. After an increase in labor supply, product and labor market tightness decrease. On the other hand, output and employment increase. In Figure 6(b), an increase in labor supply shifts outward the labor force curve, which leads to an outward shift of the labor supply curve. As a consequence, labor market tightness falls but the number of producers increases. With more producers, firms' capacity increases so the aggregate supply shifts outward in Figure 3(a). As a consequence, product market tightness falls. Since product market tightness falls, the labor demand curve rotates inward in Figure 6(b), labor market tightness falls further, and the number of producers falls back. Overall, the direct effects dominate so product and labor market tightness decrease.

Finally, we study labor market mismatch shocks. We parameterize an increase in mismatch

by a decrease in matching efficacy on the labor market along with a corresponding decrease in recruiting costs: $\hat{f}(\theta)$, $\hat{q}(\theta)$, and ρ become $\lambda \cdot \hat{f}(\theta)$, $\lambda \cdot \hat{q}(\theta)$, and $\lambda \cdot \hat{\rho}$ with $\lambda < 1$. Consequently, the function $\hat{\tau}$ remains the same. The interpretation of an increase in mismatch is that a fraction of potential workers are not suitable to employers, which reduces matching efficacy, and these unsuitable workers can be spotted at no cost, which reduces the cost of managing a vacancy.[24] After an increase in mismatch, product and labor market tightness increase. On the other hand, output and employment decrease. In Figure 6(b), an increase in mismatch shifts inward the labor supply curve but does not affect the labor demand curve. This leads to an increase in labor market tightness but a reduction in the number of producers. With fewer producers, firms' capacity decreases so the aggregate supply curve shifts inward in Figure 3(a). As a consequence, product market tightness rises. Since product market tightness rises, the labor demand curve rotates outward in Figure 6(b), labor market tightness increases further, and the number of producers rises back. Overall, the direct effects dominate so product and labor market tightness increase.

3.6 Efficient Equilibrium

We extend the definition and analysis of the efficient equilibrium to the model with product and labor market.

DEFINITION 15. *An* efficient equilibrium *consists of a product market tightness, a price, a labor market tightness, and a real wage* (x, p, θ, w) *such that supply equals demand on the product and labor markets and labor and product market tightnesses are efficient:* $c^s(x,\theta) = c^d(x,p)$, $n^s(\theta) = n^d(\theta, x, w)$, $x = x^*$, *and* $\theta = \theta^*$.

PROPOSITION 6. *There exits a unique efficient equilibrium. The efficient price and real wage are given by*

$$p^* = \chi \cdot \left\{ \frac{(1+\tau(x^*))^{1-\varepsilon}}{f(x^*)} \cdot \left[\frac{1+\hat{\tau}(\theta^*)}{\hat{f}(\theta^*)} \right]^{\alpha} \cdot \frac{\mu}{a \cdot h^{\alpha}} \right\}^{\frac{1}{\varepsilon}} \tag{16}$$

$$w^* = f(x^*) \cdot \frac{\hat{f}(\theta^*)^{\alpha-1}}{(1+\hat{\tau}(\theta^*))^{\alpha}} \cdot \alpha \cdot a \cdot h^{\alpha-1}. \tag{17}$$

[24] We represent mismatch shocks in a reduced-form way that aims to capture the main elements of a change in labor market mismatch. For microfounded models of mismatch, see for instance Shimer [2007] and Sahin et al. [2014]. Our mismatch shock has the convenient property that it does not affect the efficient labor market tightness.

Table 2: Comparative Statics in the Model of Section 3

	Effect on:			
Increase in:	Output y	Product market tightness x	Labor market tightness θ	Employment l
		A. Fixprice equilibrium		
Aggregate demand	+	+	+	+
Technology	+	−	+	+
Labor supply	+	−	−	+
Mismatch	−	+	+	−
		B. Efficient equilibrium		
Aggregate demand	0	0	0	0
Technology	+	0	0	0
Labor supply	+	0	0	+
Mismatch	−	0	0	−

Notes: An increase in aggregate demand is an increase in endowment of nonproduced good, μ, or an increase in the taste for produced good, χ. An increase in technology is an increase in the production-function parameter, a. An increase in labor supply is an increase in the number of workers in the labor force, h. An increase in mismatch is a decrease on the matching efficacy on the labor market along with a corresponding decrease in recruiting costs. With an increase in mismatch, $\hat{f}(\theta)$, $\hat{q}(\theta)$, and $\hat{\rho}$ become $\lambda \cdot \hat{f}(\theta)$, $\lambda \cdot \hat{q}(\theta)$, and $\lambda \cdot \hat{\rho}$, with $\lambda < 1$.

With efficient prices, product market tightness is $x = x^*$ and labor market tightness is $\theta = \theta^*$, where x^* satisfies $f'(x^*) = \rho$ and θ^* satisfies $\hat{f}'(\theta^*) = \hat{\rho}$. Accordingly, the tightnesses do not respond to any of the shocks that we consider, not even the labor market mismatch shock. A direct consequence is that labor utilization, $f(x)$, does not respond to these shocks either. Total employment and employment of producers increase after an increase in participation but fall after an increase in mismatch, since $l = \hat{f}(\theta) \cdot h$ and $n = \left[\hat{f}(\theta) - \hat{\rho} \cdot \theta\right] \cdot h$. The level of unemployment rises after an increase in participation and an increase in mismatch, since $u = \left[1 - \hat{f}(\theta)\right] \cdot h$ (the unemployment rate remains unchanged). Output and consumption rise after an increase in participation but fall after an increase in mismatch, since $y = f(x) \cdot a \cdot n^{\alpha}$ and $c = [f(x) - \rho \cdot x] \cdot a \cdot n^{\alpha}$. Total employment, employment of producers, and unemployment remain the same after an increase in technology. Output and consumption rise after an increase in technology.

3.7 Other Equilibria

We extend the results on the bargaining equilibrium and the equilibrium with partially rigid prices to the complete model with product and labor markets.

Bargaining Equilibrium. The expression for the equilibrium product market tightness, given by (12), remains valid. Here we derive the expression for the equilibrium labor market tightness. Following the literature on Nash bargaining, we assume that firms have a linear production function $a \cdot n$, which is the special case of our production function when $\alpha = 1$. The optimal employment choice of firms, given by (14), yields $(1 + \hat{\tau}(\theta)) \cdot w = a \cdot f(x)$. The labor demand is perfectly elastic with respect to θ. In Figure 6(b), the labor demand would be represented by an horizontal curve. The real wage is the generalized Nash solution of the bargaining problem between a firm and a worker with bargaining power $\hat{\beta} \in (0,1)$. After a match is made, the surplus to the firm of employing one worker is $\mathscr{F}(w) = a \cdot f(x) - w$, and the surplus to the worker of being employed is $\mathscr{W}(w) = w$. The Nash solution maximizes $\mathscr{F}(w)^{1-\hat{\beta}} \cdot \mathscr{W}(w)^{\hat{\beta}}$, so $\mathscr{W}(w) = \hat{\beta} \cdot [\mathscr{W}(w) + \mathscr{F}(w)] = \hat{\beta} \cdot a \cdot f(x)$ and the real wage satisfies $w = \hat{\beta} \cdot a \cdot f(x)$. Combining the labor demand condition and the bargained wage, we obtain equilibrium product market tightness:

$$\hat{\beta} \cdot (1 + \hat{\tau}(\theta)) = 1. \tag{18}$$

In equilibrium, product and labor market tightness are determined by (12) and (18). They solely depend on the matching costs and matching functions, because they are determined by the functions τ and $\hat{\tau}$. Accordingly, the comparative statics are exactly the same as those of the efficient equilibrium, even though the tightnesses may be inefficient.

Equilibrium with Partially Rigid Prices. The comparative statics in the equilibrium with partially rigid prices may not always be exactly the same as in the fixprice equilibrium. There are a number of cases to study, depending on the rigidity of the real wage relative to that of the price. We leave the characterization of all the possible cases for future research. Here we assume that the rigidity of the real wage and the price are the same. This example shows that the comparative statics of the fixprice equilibrium may also apply if price and real wage are only partially rigid.

The efficient price and real wage, given by (16) and (17), are benchmarks that define price and real wage flexibility. We define a partially rigid price and a partially rigid real wage as departures from these benchmarks. The price and real wage are given by

$$p = p_0 \cdot \chi^{\xi} \cdot \left(\frac{\mu}{a \cdot h^{\alpha}}\right)^{\frac{\xi}{\varepsilon}} \tag{19}$$

$$w = w_0 \cdot \left(\alpha \cdot a \cdot h^{\alpha-1}\right)^{\xi}. \tag{20}$$

where $\xi \in [0,1)$ parameterizes both price and real wage rigidity. We show in Appendix B that the equilibrium conditions with partially rigid prices retain the same properties as those with completely rigid prices. Therefore, all the comparative statics of the fixprice equilibrium remain valid in the equilibrium with partially rigid prices.

4 A Dynamic Model with Long-Term Relationships

We have proposed a model of unemployment relying on matching frictions on the product and the labor market. If matching frictions were an important hindrance to trade on these markets, we would expect buyers and sellers to enter long-term relationships that would alleviate these frictions.

Observations from a survey of bakers that we conducted in France in the summer of 2007 is useful to understand how long-term customer relationships alleviate matching frictions.[25] A first observation is that customer relationships alleviate the uncertainty associated with random demand. A baker in Aix-en-Provence told us that demand is difficult to predict and that having a large clientele of loyal customers who make it a habit to purchase bread in the shop was therefore important. Another baker defined his "real customers" as people who come to his shop everyday. A second observation is that customer relationships alleviate the uncertainty associated with random supply. Being a customer means having the assurance that your usual bread will be available, even on days when supply runs low. In fact, one baker said that it would be "unacceptable" to run out of bread for a regular customer, and that customers would probably "leave the bakery" if that happened. Of course, this is possible because bakers know exactly what customers order every day through their long association. It is so important to guarantee bread that our baker always has extra

[25] See Eyster, Madarasz and Michaillat [2014] for more details about the survey.

bread left over at the end of the day.

Measuring the presence of long-term relationships is therefore an indication of the prevalence of matching frictions.[26] In the data, a broad range of evidence points to the importance of these long-term relationships both on the product and labor markets, lending support to the matching framework. A number of researchers have surveyed firms and elicited the fraction of sales usually conducted with repeat customers. Figure 2(c) displays the fraction of sales to long-term customers from eleven surveys in different countries. Across these countries, the average share of sales to long-term customers is 77%, which is significant. It is also well known that most employment relationships are long lasting [for example, Hall, 1982]. For comparison, Figure 2(c) displays the share of workers in long-term employment relationships in the same countries. These shares are obtained from the OECD database for 2005. On average across the eleven countries, the share of workers in long-term employment is 87%, slightly above the share of sales to long-term customers.

For the US, there is additional evidence that seller-buyer relationships last a long time. Goldberg and Hellerstein [2011] present evidence on long-term relationships in the market for intermediate goods. Using BLS data on firms' contractual arrangements, they find that one third of all transactions are conducted under contract across industries, for both goods and services. Okun [1981] also argues that most trade on the product market occur in the context of long-term bilateral relationships. He offers the examples of the US steel and copper markets: US firms and their customers are in long-term relationships, despite the existence of a spot market for imported metal. Last, Kranton and Minehart [2001] provide compelling evidence that most goods are exchanged between buyers and sellers who already have a relationship.

Given the importance of long-term customer and employment relationships, two questions arise: how should we model the relationships? and are the results of the static model modified once the relationships are taken into account? In this section we embed the static model of Section 3 into a dynamic environment to address these questions. The dynamic model offers a better mapping between theoretical and empirical quantities, so we will use it in the empirical analysis in Section 5. The dynamic model is easy to use because at the limit without time discounting, its comparative steady states are exactly the same as the comparative statics of the model of Section 3.

[26] Of course, different models also predict the formation of long-term customer relationships. See for instance Phelps and Winter [1970], Klemperer [1987], Bils [1989], and Nakamura and Steinsson [2011].

Matching Process on the Labor and Product Markets. For tractability, we work in continuous time. Firms engage in long-term customer relationships with consumers on the product market, and they engage in long-term employment relationships with workers on the labor market. In a relationship, the buyer does not incur the matching cost and the seller sells his production for sure.

At time t, there are h workers in the labor force, $l(t)$ employed workers, and $u(t) = h - l(t)$ unemployed workers. Firms post $\hat{v}(t)$ vacancies and labor market tightness is $\theta(t) = \hat{v}(t)/u(t)$. Employment relationships are destroyed at rate $\hat{s} > 0$. Unemployed workers find a job at rate $\hat{f}(\theta(t))$ and a vacancy is filled at rate $\hat{q}(\theta(t))$. The functions $\hat{f}$ and $\hat{q}$ have the same expression as in the static model. The number of new employment relationship at time t is given by

$$\dot{l}(t) + \hat{s} \cdot l(t) = \hat{f}(\theta(t)) \cdot (h - l(t)) = \hat{q}(\theta(t)) \cdot \hat{v}(t). \tag{21}$$

The product market functions exactly as the labor market. All purchases take place through long-term customer relationships. At time t, firms have a capacity $k(t) = a \cdot n(t)^{\alpha}$ and sell output $y(t) < k(t)$. Idle capacity is $k(t) - y(t)$; this is the equivalent of unemployment, $h - l(t)$, on the labor market. Consumers create new customer-firm relationships by visiting $v(t)$ sellers that have $k(t) - y(t)$ capacity available; this is the equivalent of firms posting vacancies to hire unemployed workers on the labor market. Hence, the product market tightness is $x(t) = v(t)/(k(t) - y(t))$. The $k(t) - y(t)$ units of production available at time t are sold at rate $f(x(t))$ and the $v(t)$ visits are successful at rate $q(x(t))$. The functions f and q have the same expressions as in the static model. The customer relationships are destroyed at rate s. The number of new customer relationships at time t is given by

$$\dot{y}(t) + s \cdot y(t) = f(x(t)) \cdot (k(t) - y(t)) = q(x(t)) \cdot v(t). \tag{22}$$

Households. The utility function of the representative household is given by

$$\int_{t=0}^{+\infty} e^{-\sigma \cdot t} \cdot \left(\frac{\chi}{1+\chi} \cdot c(t)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+\chi} \cdot m(t)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} dt, \tag{23}$$

where $\sigma > 0$ is the time discount factor, $c(t)$ is consumption at time t, and $m(t)$ is holding of nonproduced good at time t. Each visit costs ρ units of good so

$$y(t) = c(t) + \frac{\rho}{q(x(t))} \cdot (\dot{y}(t) + s \cdot y(t)). \tag{24}$$

The nonproduced good does not depreciate but is taxed at rate $\zeta > 0$ by the government. This tax is a simple and crude assumption that ensures that the household consumes some produced good even when it becomes infinitely patient at the limit without time discounting.[27] The nonproduced good is an asset with law of motion

$$\dot{m}(t) = p(t) \cdot w(t) \cdot l(t) - p(t) \cdot y(t) - \zeta \cdot m(t) + T(t), \tag{25}$$

where $T(t)$ includes firms' nominal profits, which are rebated to the household, and the revenue from the tax on nonproduced good, which is transferred lump sum by the government to the household. Given $[p(t), w(t), x(t), l(t), T(t)]_{t=0}^{+\infty}$, the representative household's problem is to choose $[y(t), c(t), m(t)]_{t=0}^{+\infty}$ to maximize (23) subject to (24) and (25).

Firms. The representative firm maximizes the discounted stream of real profits

$$\int_{t=0}^{+\infty} e^{-\sigma \cdot t} \cdot (y(t) - w(t) \cdot l(t))\, dt. \tag{26}$$

The firm employs $n(t)$ producers and $l(t) - n(t)$ recruiters. Each recruiter handles $1/\hat{\rho}$ vacancy so the numbers of producers and recruiters are related by

$$l(t) = n(t) + \frac{\hat{\rho}}{\hat{q}(\theta(t))} \cdot \left(\dot{l}(t) + \hat{s} \cdot l(t)\right). \tag{27}$$

The firm sells output $y(t)$ to customers. The amount of sales depend on product market tightness and the production of the firm:

$$\dot{y}(t) = f(x(t)) \cdot (a \cdot n(t)^{\alpha} - y(t)) - s \cdot y(t). \tag{28}$$

[27] This assumption is not required in the dynamic monetary model developed in Michaillat and Saez [2014]. In that model, we can study the steady state even with zero time discounting as long as price inflation is positive.

Given $[w(t), x(t), \theta(t)]_{t=0}^{+\infty}$, the representative firm's problem is to choose $[l(t), n(t), y(t)]_{t=0}^{+\infty}$ to maximize (26) subject to (27) and (28).

Steady-state equilibrium with no time discounting. In the steady-state equilibrium with no time discounting ($\sigma \to 0$), the six variables $\{l, n, y, c, \theta, x\}$ satisfy the following six equations. Setting $\dot{l}(t) = 0$ and $\dot{y}(t) = 0$ in (21) and (22), we find that employment and output are related to tightnesses by

$$l = \frac{\hat{f}(\theta)}{\hat{s} + \hat{f}(\theta)} \cdot h \tag{29}$$

$$y = \frac{f(x)}{s + f(x)} \cdot a \cdot n^{\alpha}. \tag{30}$$

Setting $\dot{l}(t) = 0$ and $\dot{y}(t) = 0$ in (27) and (24), we find that number of producers and consumption are related to employment and output by

$$l = (1 + \hat{\tau}(\theta)) \cdot n$$

$$y = (1 + \tau(x)) \cdot c,$$

where $\tau(x) \equiv s \cdot \rho / (q(x) - s \cdot \rho)$ and $\hat{\tau}(\theta) \equiv \hat{s} \cdot \hat{\rho} / (\hat{q}(\theta) - \hat{s} \cdot \hat{\rho})$ are the same functions as in the static model once the parameters ρ and $\hat{\rho}$ are replaced by the parameters $s \cdot \rho$ and $\hat{s} \cdot \hat{\rho}$. Appendix C solves the optimal control problems of the representative household and firm. In steady state, the optimal employment decision of the firm is

$$n = \left[\frac{\alpha \cdot a}{(1 + \hat{\tau}(\theta)) \cdot w} \cdot \frac{f(x)}{s + f(x)} \right]^{\frac{1}{1-\alpha}}, \tag{31}$$

and the optimal consumption decision of the household combined with the market-clearing condition for the nonproduced good yields

$$c = \left[\frac{\chi \cdot \zeta}{(1 + \tau(x)) \cdot p} \right]^{\varepsilon} \cdot \mu.$$

These equations describe the employment, output, labor supply, aggregate supply, labor demand, and aggregate demand curves. These curves are exactly those of the static model of Sec-

tion 3 once χ, $f(x)$, and $\hat{f}(\theta)$ are replaced by $\chi \cdot \zeta$, $f(x)/(s+f(x))$, and $\hat{f}(\theta)/(\hat{s}+\hat{f}(\theta))$. All the relevant properties of the functions f and $\hat{f}$ are preserved by the transformation to $f/(s+f)$ and $\hat{f}/(\hat{s}+\hat{f})$. Therefore, the comparative steady states of the dynamic model are exactly the same as the comparative statics of the model of Section 3.

5 Exploration of the Sources of Labor Market Fluctuations

In this section we use the dynamic model to re-examine the origins of labor market fluctuations. To isolate fluctuations at business-cycle frequency, we remove from all the quarterly time series a low frequency trend—a Hodrick-Prescott (HP) filter with smoothing parameter 1600. Our strategy is to compare the empirical correlations between the cyclical component of several variables with the theoretical correlations obtained by comparative steady states for various shocks.[28] Through the lens of the model, the empirical evidence suggests that price and real wage are somewhat rigid, and that labor market fluctuations are mainly driven by aggregate demand shocks.

5.1 A Proxy for Product Market Tightness and Other Variables

The empirical analysis relies on the empirical behavior of product market tightness. We do not know of any series of product market tightness. We therefore construct a proxy for the cyclical fluctuations of product market tightness, x_t, for the US. Our proxy is the cyclical fluctuations of labor utilization, $f(x_t)/(s+f(x_t))$. Labor utilization is just one minus the share of idle time. Fluctuations of labor utilization are a good proxy as long as the function f and the separation rate s are stable at business-cycle frequency.

We could use the measure of labor utilization constructed by the Institute for Supply Management (ISM) based on a survey that they administer. The survey elicits the actual production level of firms as a share of their maximum production level given their current capital stock and workforce, which exactly corresponds to our concept of labor utilization. However, their measure of

[28] This empirical strategy is borrowed from the seminal study of Blanchard and Diamond [1989*b*]. To separate between three types of labor market shocks (aggregate activity, reallocation, and labor force participation), they compare the correlations predicted by their model with empirical correlations. Blanchard and Diamond [1989*b*] also confirm the robustness of the correlation results by running vector autoregressions (VAR). Here we focus on empirical correlations and leave the VAR analysis for future work as the VAR analysis requires additional identifying assumptions.

labor utilization is only available from 1989 to 2013 for the manufacturing sector and from 1999 to 2013 for the service sector. We would like to obtain a longer product market tightness series, so we resort to another source of data. To obtain a longer series, we use the capacity utilization measure cu_t constructed by the Census Bureau from the Survey of Plant Capacity (SPC) for 1973–2013.[29] Figure 2(a) plots this measure.[30] The measure applies to the manufacturing sector.[31]

The measure cu_t is based on capacity at full employment, whereas our concept of labor utilization is based on capacity at current employment.[32] We therefore correct cu_t to obtain a measure of labor utilization. Let $g(a,n,K) = a \cdot n^{\alpha} \cdot K^{1-\alpha}$ be a firm's production function under technology a, employment n, and capital stock K. The capacity of the firm under current employment is $g(a,n,K)$. The capacity of the firm under full employment is $g(a,N,K)$, where N is the level of full employment that respondents have in mind when they report cu. We assume that N like K are fixed over time, which is a valid assumption at business-cycle frequency if N and K move much more slowly than n. Using the definition of capacity utilization in the SPC and equation (30), we write

$$y_t = cu_t \cdot g(a_t,N,K) = \frac{f(x_t)}{s+f(x_t)} \cdot g(a_t,n_t,K),$$

which yields an expression for labor utilization:

$$\frac{f(x_t)}{s+f(x_t)} = cu_t \cdot \frac{g(a_t,N,K)}{g(a_t,n_t,K)} = cu_t \cdot \left(\frac{N}{n_t}\right)^{\alpha}.$$

[29] There are other measures of capacity utilization. We choose the measure of the Census Bureau because it is available for the longest period and uses the broadest sample of firms. The Federal Reserve Board also constructs a measure of capacity utilization for the industrial sector since 1967, but their measure is difficult to use because it combines several measures of capacity utilization (including the measure of the Census Bureau) and industrial production data [Morin and Stevens, 2004].

[30] Until 2006, the SPC measures fourth-quarter capacity utilization rates. From 2008 to 2013, the SPC measures the capacity utilization at quarterly frequency. To obtain a quarterly series for 1973–2007, we use a linear interpolation of the annual series into a quarterly series. We combine this interpolated series with the quarterly series for 2008–2013.

[31] Our measure of capacity utilization does not apply to the service sector. Available evidence suggests, however, that fluctuations in labor utilization in the service sector may be similar to those in the manufacturing sector. The ISM measures labor utilization in each sector, and the cyclical fluctuations of the two series are highly correlated. Over the 1999:Q4–2013:Q2 period, the correlation is 0.77.

[32] Morin and Stevens [2004] explain that "the capacity indexes [from the SPC] are designed to embody the concept of sustainable practical capacity, defined as the greatest level of output each plant in a given industry can maintain within the framework of a realistic work schedule, taking account of normal downtime and assuming sufficient availability of inputs to operate machinery and equipment in place."

This equation holds at any time t. Taking log and first differences yields

$$\Delta \ln \left(\frac{f(x_t)}{s + f(x_t)} \right) = \Delta \ln(cu_t) - \alpha \cdot \Delta \ln(n_t).$$

We measure n_t as the quarterly average of the seasonally adjusted monthly employment level in the manufacturing sector constructed by the BLS Current Employment Statistics (CES) program. Following conventions, we set the production-function parameter to $\alpha = 2/3$. Using the first differences of $\ln(cu_t)$ and $\ln(n_t)$, we obtain the first difference of $\ln[f(x_t)/(s+f(x_t))]$. We sum these first differences to obtain a series for $\ln[f(x_t)/(s+f(x_t))]$. We detrend the series to obtain its cyclical fluctuations.[33] This is our proxy for the cyclical fluctuations of product market tightness. Figure 7(a) plots the proxy for 1974:Q1–2013:Q2.

The empirical analysis also requires measures of output, employment, and labor market tightness. We also construct them for 1974:Q1–2013:Q2. All the data are for the US and seasonally adjusted. We measure output and employment using quarterly indices for real output and employment for the nonfarm business sector constructed by the MSPC program of the BLS. We construct labor market tightness as the ratio of vacancy to unemployment. We measure unemployment with the quarterly average of the monthly unemployment level constructed by the BLS from the CPS. We measure vacancy with the quarterly average of the monthly vacancy index constructed by Barnichon [2010]. This index combines the online and print help-wanted indices of the Conference Board.[34] We divide vacancy by unemployment to obtain a series for labor market tightness.

5.2 Evidence of Rigid Prices

Before assessing the predictions of the model, we evaluate whether flexible prices or rigid prices offer a better description of the data. As highlighted in Table 2, there is one main difference between the two assumptions. With rigid prices, labor and product market tightnesses fluctuate in response to demand and supply shocks. With flexible prices—either efficient or bargained—labor

[33] The measure of labor utilization that we construct is highly correlated with the ISM measure of labor utilization for manufacturing. Over the 1989:Q4–2013:Q2 period, when the two series overlap, the correlation of their cyclical fluctuations is 0.68.

[34] The Conference Board indices are a standard measure of vacancy. Another common measure is the vacancy index constructed by the BLS from the Job Opening and Turnover Survey (JOLTS). We cannot use this other measure here because the JOLTS begins only in December 2000 and does therefore not provide a long enough series for our analysis.

and product market tightnesses do not fluctuate in response to shocks.

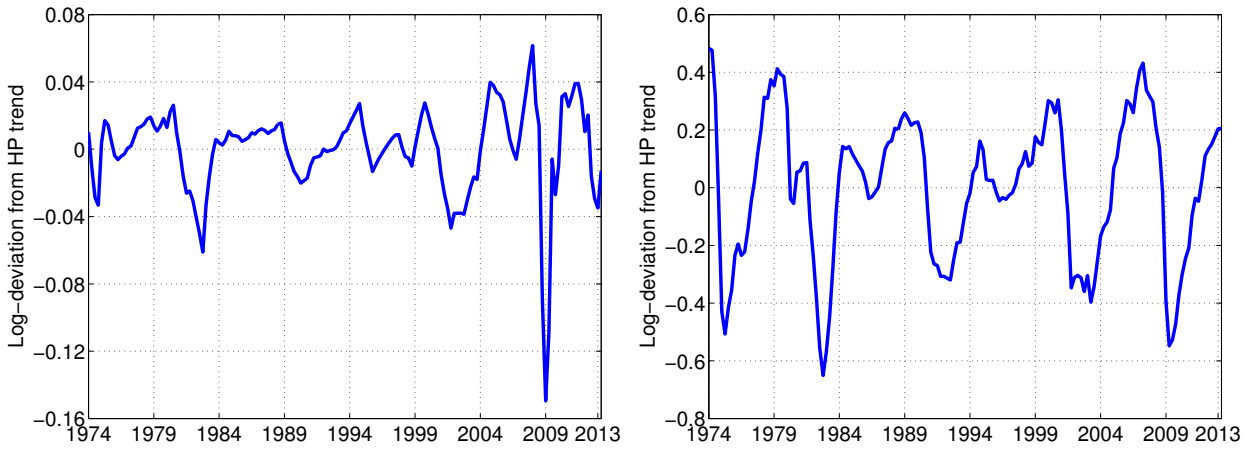
Figure 7 displays the cyclical fluctuations of labor and product market tightnesses. The figure confirms the well-known fact that labor market tightness is subject to large fluctuations over the business cycle [for example, Blanchard and Diamond, 1989*b*; Shimer, 2005]. The standard deviation of detrended log labor market tightness is 25.0%. The standard deviation of detrended log product market tightness is 2.6%. The two standard deviations are not directly comparable, however. We do not observe product market tightness directly but a proxy that moves together with product market tightness but with a different amplitude. Translating the 2.6% into the actual standard deviation of product market tightness would require information about the product market matching function.

The observed cyclical fluctuations of product and labor market tightness are significant. In the context of our model, this result suggests that an equilibrium with partially rigid prices is more appropriate to describe business-cycle fluctuations than an equilibrium with fully flexible prices. It also suggests that business-cycle fluctuations are inefficient because efficiency implies constant tightnesses. This result is related to the finding of Shimer [2005] and Hall [2005] that the large fluctuations of labor market tightness observed in US data indicate some real wage rigidity.

In the empirical analysis below, we exploit the predictions arising from the equilibrium with completely or partially rigid prices. These predictions are reported in panel A of Table 2.

5.3 Evidence of Labor Demand Shocks

In our model, labor supply shocks and mismatch shocks are the only shocks that generate a negative correlation between labor market tightness and employment. All labor demand shocks, either aggregate demand or technology shocks, produce a positive correlation. In Figure 8(a), we plot together the cyclical fluctuations in labor market tightness and employment to determine their correlation. Fluctuations in labor market tightness and employment appear strongly positively correlated. Figure 8(b) formalizes this observation. It displays the cross-correlogram of labor market tightness and employment. Labor market tightness leads employment by one lag. At one lag, the correlation of recruiting wedge and employment is large, 0.95. The contemporaneous correlation is broadly the same, 0.93.



(a) Proxy for product market tightness

(b) Labor market tightness

Figure 7: Cyclical Fluctuations of Product Market Tightness and Labor Market Tightness

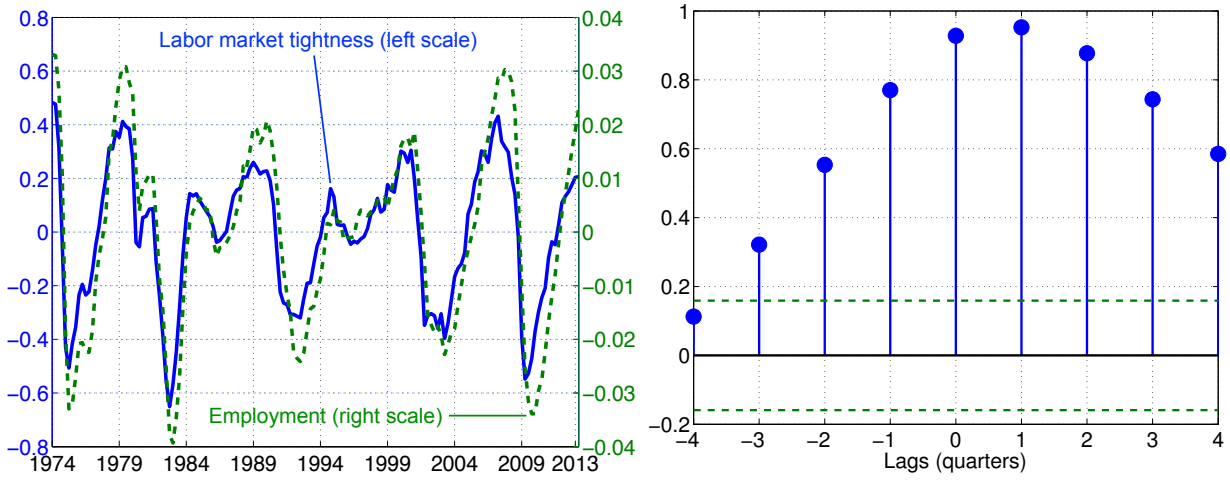
Notes: The time period is 1974:Q1–2013:Q2. The proxy for product market tightness is constructed as explained in the text. Labor market tightness is constructed as $\theta_t = v_t/u_t$, where v_t is the quarterly average of the monthly vacancy index constructed by Barnichon [2010], and u_t is the quarterly average of the seasonally adjusted monthly unemployment level constructed by the BLS from the CPS. Panels (a) and (b) report detrended product market tightness and labor market tightness. The detrended series are constructed by taking the log of the series and removing the trends produced by a HP filter with smoothing parameter 1600.

These high correlations are strong evidence that labor demand and not labor supply or mismatch shocks generate labor market fluctuations. This result confirms with more recent data the result from the classical analysis of Blanchard and Diamond [1989*b*].[35]

5.4 Evidence of Aggregate Demand Shocks

An aggregate demand shock is the only shock that generates a positive correlation between product market tightness and output. A technology shock produces a negative correlation. Here we study the correlation between product market tightness and output to assess the relative importance of aggregate demand and technology shocks. In Figure 9(a), we plot together the cyclical fluctuations in product market tightness and output. It does seem, on balance, that fluctuations in product market tightness and output are positively correlated. This finding suggests that aggregate demand shocks but not technology shocks are the main source of labor market fluctuations at business-

[35] We use the property that a positive correlation between labor market tightness and employment indicates labor demand shocks. Blanchard and Diamond [1989*b*] use instead the equivalent property that a negative correlation between vacancy rate and unemployment rate, materialized by a movement along the Beveridge curve, indicates labor demand shocks.



(a) Log-deviations from HP trend

(b) Cross-correlogram (tightness leads)

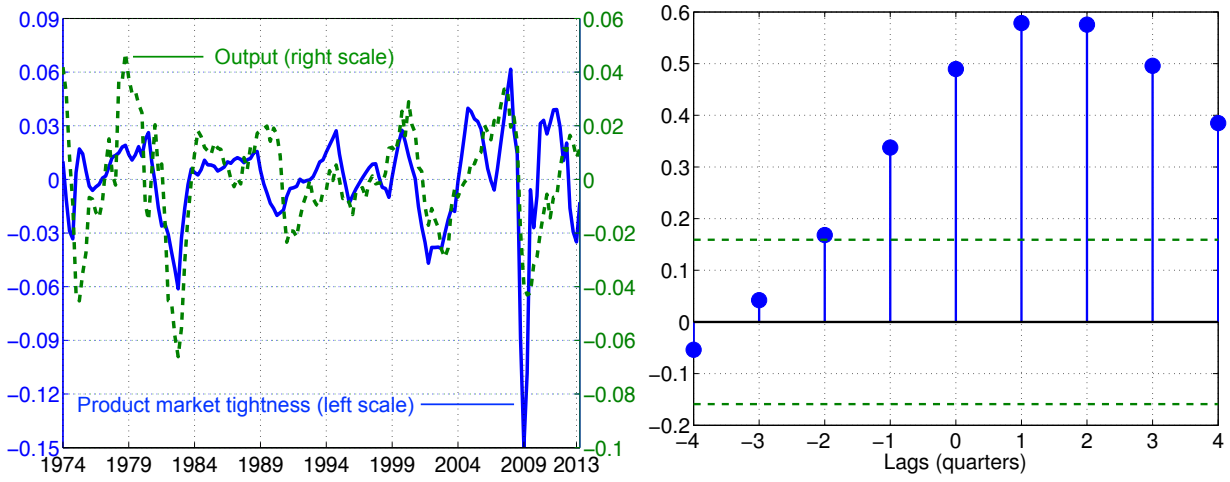
Figure 8: Correlation Between Labor Market Tightness and Employment

Notes: The time period is 1974:Q1–2013:Q2. Labor market tightness is constructed as $\theta_t = v_t/u_t$, where v_t is the quarterly average of the monthly vacancy index constructed by Barnichon [2010], and u_t is the quarterly average of the seasonally adjusted monthly unemployment level constructed by the BLS from the CPS. Employment, l_t, is the seasonally adjusted quarterly index for employment in the nonfarm business sector constructed by the BLS MSPC program. Panel (a) reports detrended labor market tightness and employment. The detrended series are constructed by taking the log of the series and removing the trends produced by a HP filter with smoothing parameter 1600. Panel (b) reports the cross-correlogram of the two detrended series. The cross-correlation at lag i is the correlation between θ_{t-i} and l_t.

cycle frequency. In particular, the large output drops in 1981–1982, 2001, and 2008–2009 were accompanied by large drops in product market tightness, suggesting that these recessions were caused by a negative aggregate demand shock. There are some exceptions, however. From 2004 to 2006, output was increasing while product market tightness was falling. This observation is consistent with a positive technology shock in the 2004–2006 period.

Figure 9(b) formalizes the observation that product market tightness and output are positively correlated. It displays the cross-correlogram of product market tightness and real output. Product market tightness leads output by one lag. At one lag, the correlation of product market tightness and output is quite large, 0.59; the contemporaneous correlation is 0.49. These correlations are statistically significant.

The implication of the positive correlation between product market tightness and output is that aggregate demand shocks but not technology shocks are the main source of labor market fluctuations at business-cycle frequency. Our work confirms with a different empirical strategy the finding of Galí [1999], Shea [1998], and Basu, Fernald and Kimball [2006] that technology shocks



(a) Log-deviations from HP trend

(b) Cross-correlogram (tightness leads)

Figure 9: Correlation Between Product Market Tightness and Output

Notes: The time period is 1974:Q1–2013:Q2. Output, y_t, is the seasonally adjusted quarterly index for real output in the nonfarm business sector constructed by the BLS MSPC program. Product market tightness, x_t, is constructed as explained in the text. Panel (a) reports detrended product market tightness and output. The detrended series are constructed by taking the log of the series and removing the trends produced by a HP filter with smoothing parameter 1600. Panel (b) reports the cross-correlogram of the two detrended series. The cross-correlation at lag i is the correlation between x_{t-i} and y_t.

are unlikely to be the main source of business cycles.

Given that measured productivity is procyclical, the result that technology shocks do not explain a large share of business-cycle fluctuations may seem puzzling.[36] Indeed, output fluctuates more than employment over the business cycle so that labor appears more productive in booms than in recessions, and this fact motivated the development of Real Business Cycle models in which technology drives macroeconomic fluctuations [Prescott, 2006].

The presence of matching frictions on the product market reconciles the observations that aggregate demand shocks are a major source of fluctuations and that measured productivity is procyclical.[37] Matching frictions on the product market make it difficult for firms to find customers and prevent firms from fully utilizing their labor. As a consequence, firms' employees are idle part of the time. In this context, an increase in aggregate demand raises product market tightness,

[36] Galí and van Rens [2010] show that the correlation of output and labor productivity, measured as the output-employment ratio, is positive over the 1948–2007 period, even though it decreased over the period. They show that the correlation is around 0.8 for the 1948–1983 period and around 0.5 for the 1984–2007 period. Both correlations are significantly different from zero. See Table 1, panel B, columns 1 and 2 in Galí and van Rens [2010].

[37] Bai, Rios-Rull and Storesletten [2012] have studied the response of measured productivity to aggregate demand shocks in a model with matching frictions on the product market and have reached similar conclusions.

which reduces idle time for firms' employees and increases measured labor productivity. Although they are based on a different empirical strategy, our results are consistent with the finding of Basu [1996] that cyclical fluctuations in measured productivity are driven mostly by variable utilization of labor and capital and not by technology shocks.

6 Conclusion

This paper develops a tractable model of unemployment fluctuations. The model can be seen as an equilibrium version of the model of Barro and Grossman [1971]. To explore the sources of labor market fluctuations, we compare the theoretical correlations from the model with the corresponding empirical correlations. The analysis suggests that these fluctuations are driven by aggregate demand shocks that shift the labor demand in the presence of price and real wage rigidities.

Our model could be useful to study various policies that affect at the same time labor supply, labor demand, and aggregate demand. One such policy is unemployment insurance. Increasing unemployment insurance affects labor supply because it reduces job search, labor demand because it increases wages through bargaining, and aggregate demand by redistributing income from employed workers to unemployed workers with a higher marginal propensity to consume.[38] Our model offers a framework in which all these effects can be considered simultaneously. Another policy is the income tax. Making the income tax more progressive affects the labor supply by discouraging work and the aggregate demand by redistributing income from richer workers to poorer workers with a higher marginal propensity to consume. Yet another policy is the payroll tax. Shifting the payroll tax from employers to employees affects the labor demand by reducing labor costs and the aggregate demand by redistributing income from employed workers to firm owners with a lower marginal propensity to consume. Finally, the minimum wage is subject to broadly the same policy trade-offs as a payroll tax and could also be analyzed with our model. Addressing some of these questions requires to introduce heterogeneity among workers and across workers and firm owners, an extension we leave for future research.

[38] Landais, Michaillat and Saez [2010] examine this issue but do not incorporate the aggregate demand channel.

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Appendix A: Proofs

Proof of Proposition 1. The proof follows directly from the properties of the aggregate supply listed in Definition 1. Note that we can rewrite $f'(x^*) = \rho$ as $x \cdot f'(x)/f(x) = \rho/q(x)$, since $f(x)/x = q(x)$. Given the definitions of $1-\eta(x)$ and τ, this equation says that $1-\eta(x) = \tau(x)/(1+\tau(x))$, which can be rewritten as (9).

Proof of Proposition 2. In a fixprice equilibrium parameterized by $p_0 > 0$, the tightness x satisfies $c^s(x) = c^d(x, p_0)$, which is equivalent to

$$\frac{1}{1+\tau(x)} \cdot \left[f(x) \cdot k - \frac{\chi^\varepsilon \cdot \mu}{p_0^\varepsilon} \cdot \frac{1}{(1+\tau(x))^{\varepsilon-1}} \right] = 0. \tag{A1}$$

We are looking for $x \in [0, x^m]$ that solves this equation.

The function $1/(1+\tau)$ is smooth and strictly decreasing on $[0, x^m]$ with $1/(1+\tau(0)) = 1-\rho$ and $1/(1+\tau(x^m)) = 0$. Since $1/(1+\tau(x^m)) = 0$, x^m is always a solution. But there is no consumption at x^m because $c^s(x^m)) = 0$. Hence, we focus on $x \in [0, x^m)$.

As $1/(1+\tau(x)) > 0$ for $x < x^m$, $x \in [0, x^m)$ solves (A1) iff it solves

$$(1+\tau(x))^{\varepsilon-1} \cdot f(x) = \frac{\mu \cdot \chi^\varepsilon}{k \cdot p_0^\varepsilon}. \tag{A2}$$

Since $\varepsilon > 1$, the function $x \mapsto (1+\tau(x))^{\varepsilon-1} \cdot f(x)$ is strictly increasing from 0 to $+\infty$ on $[0, x^m)$. Thus, there is a unique $x \in (0, x^m)$ that solves (A2). For any $x \in (0, x^m)$, $c^s(x) > 0$. Thus, there exists a unique fixprice equilibrium parameterized by $p_0 > 0$ with positive consumption.

Proof of Proposition 3. In an efficient equilibrium, the price p satisfies $c^s(x^*) = c^d(x^*, p)$, which is equivalent to

$$(1+\tau(x^*))^{\varepsilon-1} \cdot f(x^*) \cdot \frac{k}{\mu \cdot \chi^\varepsilon} \cdot = p^{-\varepsilon}. \tag{A3}$$

We obtain (A3) in the same way as we obtain (A2). In particular, we use the fact that $x^* < x^m$ such that $1/(1+\tau(x^*)) > 0$. The left-hand-side term is positive, so there exists a unique $p > 0$ that solves (A3). Thus, there exists a unique efficient equilibrium. The price in the efficient equilibrium clearly satisfies (11).

Proof of Proposition 4. The proof is similar to that of Proposition 1.

Proof of Proposition 5. In a fixprice equilibrium parameterized by $p_0 > 0$ and $w_0 > 0$, (x, θ) satisfies $n^s(\theta) = n^d(\theta, x, w_0)$ and $c^s(x, \theta) = c^d(x, p_0)$.

We can show that there exists two equilibria with zero consumption. In one equilibrium, $x = x^m$ and $\theta = \theta^m$, such that $c = 0$ and $n = 0$. In the other equilibrium, $x = x^m$ but $\theta < \theta^m$, such that $c = 0$ but $n > 0$. In this proof, we focus on equilibria with positive consumption.

We focus on the case with $\theta < \theta^m$ and $x < x^m$. $1/(1+\tau(x)) > 0$ and $1/(1+\hat{\tau}(\theta)) > 0$ so we can rewrite the system of equilibrium conditions. The equation $n^s(\theta) = n^d(\theta, x, w_0)$ is equivalent to

$$\left(\frac{1}{1+\hat{\tau}(\theta)}\right)^{1-\alpha} \cdot \left[\hat{f}(\theta)^{1-\alpha} \cdot h^{1-\alpha} - \frac{\alpha \cdot a}{w_0} \cdot f(x) \cdot \frac{1}{(1+\hat{\tau}(\theta))^{\alpha}}\right] = 0.$$

Since the first factor is positive, this equation implies that the second factor must be zero. Multiplying the second factor by $(1+\hat{\tau}(\theta))^{\alpha}$ yields

$$h^{1-\alpha} \cdot \hat{f}(\theta)^{1-\alpha} \cdot (1+\hat{\tau}(\theta))^{\alpha} = f(x) \cdot \frac{\alpha \cdot a}{w}. \tag{A4}$$

Following the proof of Proposition 2, we modify $c^s(x,\theta) = c^d(x, p_0)$ to obtain

$$f(x) \cdot a \cdot \left(\frac{\hat{f}(\theta)}{1+\hat{\tau}(\theta)} \cdot h\right)^{\alpha} \cdot (1+\tau(x))^{\varepsilon-1} = \chi^{\varepsilon} \cdot \frac{\mu}{p^{\varepsilon}}.$$

Multiplying both sides of the equation by α/w and substituting (A4) into this equation yields

$$h \cdot \hat{f}(\theta) \cdot (1+\tau(x))^{\varepsilon-1} = \frac{\alpha}{w} \cdot \chi^{\varepsilon} \cdot \frac{\mu}{p^{\varepsilon}}. \tag{A5}$$

We show that for any $p_0 > 0$ and $w_0 > 0$, the system (A4)–(A5) admits a unique solution. Since $\alpha < 1$, the function $\theta \mapsto \hat{f}(\theta)^{1-\alpha} \cdot (1+\hat{\tau}(\theta))^{\alpha}$ is strictly increasing from 0 to $+\infty$ on $[0, \theta^m)$. Hence, equation (A4) implicitly defines θ as a function of $x \in [0, +\infty)$: $\theta = \Theta^L(x)$. In addition, f is strictly increasing from 0 to 1 on $[0, +\infty)$; therefore, Θ^L is strictly increasing on $[0, +\infty)$, $\Theta^L(0) = 0$, and $\lim_{x\to+\infty} \Theta^L(x) = \theta^L > 0$ where $\theta^L \in (0, \theta^m)$ is implicitly defined by $h^{1-\alpha} \cdot \hat{f}(\theta^L)^{1-\alpha} \cdot \left(1+\hat{\tau}(\theta^L)\right)^{\alpha} = \alpha \cdot a/w$.

If $[\alpha/(w \cdot h)] \cdot \chi^{\varepsilon} \cdot (\mu/p^{\varepsilon}) \geq 1$, define $x^P(p,w)$ by

$$\left(1+\tau(x^P)\right)^{\varepsilon-1} = \frac{\alpha}{w \cdot h} \cdot \chi^{\varepsilon} \cdot \frac{\mu}{p^{\varepsilon}}.$$

If $[\alpha/(w \cdot h)] \cdot \chi^{\varepsilon} \cdot (\mu/p^{\varepsilon}) < 1$, $x^P(p,w) \equiv 0$. Since $\varepsilon > 1$, the function $x \mapsto (1+\tau(x))^{\varepsilon-1}$ is strictly increasing from 1 to $+\infty$ on $[0, x^m)$; therefore, x^P is well defined and $x^P(p,w) \in (0, x^m)$. Since $\hat{f}$ is strictly increasing from 0 to 1 on $(0, +\infty)$, equation (A5) implicitly defines θ as a function of $x \in (x^P(p,w), x^m)$: $\theta = \Theta^P(x)$. Moreover, Θ^P is strictly decreasing on $(x^P(p,w), x^m)$, $\lim_{x\to x^P(p,w)} \Theta^P(x) = +\infty$, and $\lim_{x\to x^m} \Theta^P(x) = 0$.

The system (A4)–(A5) is equivalent to

$$\begin{cases} \Theta^L(x) &= \Theta^P(x) \\ \theta &= \Theta^P(x) \end{cases}$$

Given the properties of the functions Θ^L and Θ^P, we conclude that this system admits a unique solution (x, θ) with $x \in (x^P(p,w), x^m)$ and $\theta \in (0, \theta^L)$.

Proof of Proposition 6. In an efficient equilibrium, p and w satisfy $n^s(\theta^*) = n^d(\theta^*, x^*, w)$ and $c^s(x^*, \theta^*) = c^d(x^*, p)$. These equations are equivalent to

$$w = (1+\hat{\tau}(\theta^*))^{-\alpha} \cdot \hat{f}(\theta^*)^{\alpha-1} \cdot f(x^*) \cdot \alpha \cdot a \cdot h^{\alpha-1}$$
$$p^{\varepsilon} = \frac{(1+\tau(x^*))^{1-\varepsilon}}{\hat{f}(\theta^*)} \cdot \frac{\alpha}{w} \cdot \frac{\chi^{\varepsilon} \cdot \mu}{h}.$$

We obtain this system in the same way as we obtain (A4)–(A5). Of course, there exists a unique $w > 0$ and $p > 0$ that solves this system. Thus, there exists a unique efficient equilibrium. The real wage in the efficient equilibrium clearly satisfies (17). Combining these two equations, we find that the price in the efficient equilibrium satisfies (16).

Appendix B: Comparative Statics

This appendix derives the comparative statics for the fixprice equilibrium of the model of Section 3.

Aggregate Demand Shocks. We parameterize an increase in aggregate demand by an increase in χ or μ. To compute the comparative statics, we manipulate the two equilibrium conditions. First, we rewrite the condition $n^s(\theta) = n^d(\theta, x, w)$ as

$$f(x) \cdot a \cdot \left(\frac{\hat{f}(\theta)}{1+\hat{\tau}(\theta)} \right)^{\alpha} \cdot h^{\alpha} = \hat{f}(\theta) \cdot \frac{w \cdot h}{\alpha}. \tag{A6}$$

Rearranging terms in this condition yields

$$F(\theta, x, a, h) \equiv f(x) - \hat{f}(\theta)^{1-\alpha} \cdot (1+\hat{\tau}(\theta))^{\alpha} \cdot \frac{w \cdot h^{1-\alpha}}{\alpha \cdot a} = 0. \tag{A7}$$

The function F satisfies $\partial F/\partial \theta < 0$, $\partial F/\partial x > 0$, $\partial F/\partial a > 0$, and $\partial F/\partial h < 0$. Using the implicit function theorem, we write the solution θ to $F(\theta, x, a, h) = 0$ as a function $\Theta^F(x, a, h)$ with $\partial \Theta^F/\partial x > 0$, $\partial \Theta^F/\partial a > 0$, and $\partial \Theta^F/\partial h < 0$. Second, we combine the condition $c^s(x, \theta) = c^d(x, p)$ with (A6) to obtain

$$G(\theta, x, h, \chi, \mu) \equiv \hat{f}(\theta) \cdot (1+\tau(x))^{\varepsilon-1} - \frac{\alpha}{w \cdot h} \cdot \frac{\chi^{\varepsilon} \cdot \mu}{p^{\varepsilon}} = 0. \tag{A8}$$

The function G satisfies $\partial G/\partial \theta > 0$, $\partial G/\partial x > 0$, $\partial G/\partial h > 0$, $\partial G/\partial \chi < 0$, and $\partial G/\partial \mu < 0$. Using the implicit function theorem, we write the solution θ to $G(\theta, x, h, \chi, \mu) = 0$ as a function $\Theta^G(x, h, \chi, \mu)$ with $\partial \Theta^G/\partial x < 0$, $\partial \Theta^G/\partial h < 0$, $\partial \Theta^G/\partial \chi > 0$, and $\partial \Theta^G/\partial \mu > 0$.

Equilibrium product market tightness satisfies $G(\Theta^F(x, a, h), x, h, \chi, \mu) = 0$. Given that $\partial \Theta^F/\partial x > 0$, $\partial G/\partial \theta > 0$, $\partial G/\partial x > 0$, and $\partial G/\partial \chi < 0$, the implicit function theorem implies that $\partial x/\partial \chi > 0$. We can show similarly that $\partial x/\partial \mu > 0$. Hence, after an increase in aggregate demand, product market tightness increases. Since $\theta = \Theta^F(x, a, h)$ with $\partial \Theta^F/\partial x > 0$, labor market tightness also increases. Equation (A6) implies that $y = \hat{f}(\theta) \cdot h \cdot w/\alpha$, so output increases as well. Since labor market tightness increases, it is clear that employment increases.

Technology Shocks. We parameterize an increase in technology by an increase in a. Equilibrium product market tightness satisfies $F(\Theta^G(x,h,\chi,\mu),x,a,h)=0$. Given that $\partial\Theta^G/\partial x<0$, $\partial F/\partial\theta<0$, $\partial F/\partial x>0$, and $\partial F/\partial a>0$, the implicit function theorem implies that $\partial x/\partial a<0$. Hence, after an increase in technology, product market tightness decreases. Since $\theta=\Theta^G(x,h,\chi,\mu)$ with $\partial\Theta^G/\partial x<0$, labor market tightness increases. The logic presented for aggregate demand shocks implies that since labor market tightness increases, output and employment increase.

Labor Supply Shocks. We parameterize an increase in labor supply by an increase in h. The functions F and G both respond to h, which makes it impossible to obtain the comparative statics for x and θ. Hence, we manipulate (A7) and (A8) to be able to obtain these comparative statics. Exponentiating (A8) to the power of $1-\alpha$ and dividing (A7) by the resulting equation yields

$$H(\theta,x)\equiv(1+\hat{\tau}(\theta))^{\alpha}-f(x)\cdot(1+\tau(x))^{(1-\alpha)\cdot(\varepsilon-1)}\cdot a\cdot\left(\frac{\alpha}{w}\right)^{\alpha}\cdot\left(\frac{\chi^{\varepsilon}\cdot\mu}{p^{\varepsilon}}\right)^{\alpha-1}=0. \tag{A9}$$

The function H satisfies $\partial H/\partial\theta>0$ and $\partial H/\partial x<0$. The function H does not depend on h, which resolves our earlier problem. Using the implicit function theorem, we write the solution θ to $H(\theta,x)=0$ as a function $\Theta^H(x)$ with $\partial\Theta^H/\partial x>0$.

Equilibrium product market tightness satisfies $G(\Theta^H(x),x,h)=0$. Given that $\partial\Theta^H/\partial x>0$, $\partial G/\partial\theta>0$, $\partial G/\partial x>0$, and $\partial G/\partial h>0$, the implicit function theorem implies that $\partial x/\partial h<0$. Hence, after an increase in labor supply, product market tightness decreases. Since $\theta=\Theta^H(x)$ with $\partial\Theta^H/\partial x>0$, labor market tightness also decreases. Output is given by $y=(1+\tau(x))\cdot c=(1+\tau(x))^{1-\varepsilon}\cdot\mu\cdot\chi^{\varepsilon}/p^{\varepsilon}$ and $1-\varepsilon<0$; therefore, output increases when x decreases. Employment is given by $l=(1+\hat{\tau}(\theta))\cdot n=(1+\hat{\tau}(\theta))\cdot^{-\alpha/(1-\alpha)}\cdot(f(x)\cdot a\cdot\alpha/w)^{1/(1-\alpha)}$ and $-\alpha/(1-\alpha)<0$; therefore, employment increases when θ decreases.

Mismatch Shocks. We parameterize an increase in mismatch by a decrease in matching efficacy on the labor market along with a corresponding decrease in recruiting costs: $\hat{f}(\theta)$, $\hat{q}(\theta)$, and ρ become $\lambda\cdot\hat{f}(\theta)$, $\lambda\cdot\hat{q}(\theta)$, and $\lambda\cdot\hat{\rho}$ with $\lambda<1$. Consequently, the function $\hat{\tau}$ remains the same. With the parameter λ for mismatch, the functions H and Θ^H are the same, but the function G depends on λ with $\partial G/\partial\lambda>0$. Equilibrium product market tightness satisfies $G(\Theta^H(x),x,\lambda)=0$. Given that $\partial\Theta^H/\partial x>0$, $\partial G/\partial\theta>0$, $\partial G/\partial x>0$, and $\partial G/\partial\lambda>0$, the implicit function theorem implies that $\partial x/\partial\lambda<0$. Hence, after an increase in mismatch, product market tightness increases. Since $\theta=\Theta^H(x)$ with $\partial\Theta^H/\partial x>0$, labor market tightness also increases. The logic presented for labor supply shocks implies that both output and employment fall after an increase in mismatch.

Using these expressions for the partially rigid real wage and price, (20) and (19), we can rewrite

equations (A7), (A8), and (A9). We find that these equations become

$$0 = f(x) - \hat{f}(\theta)^{1-\alpha} \cdot (1+\hat{\tau}(\theta))^{\alpha} \cdot \frac{w_0 \cdot h^{(1-\alpha)\cdot(1-\xi)}}{(\alpha \cdot a)^{1-\xi}}$$

$$0 = \hat{f}(\theta) \cdot (1+\tau(x))^{\varepsilon-1} - \frac{\alpha}{w_0 \cdot h^{1-\xi}} \cdot \frac{(\chi^{\varepsilon} \cdot \mu)^{1-\xi}}{p_0^{\varepsilon}}$$

$$0 = (1+\hat{\tau}(\theta))^{\alpha} - f(x) \cdot (1+\tau(x))^{(1-\alpha)\cdot(\varepsilon-1)} \cdot a^{1-\xi} \cdot \frac{\alpha^{\alpha-\xi}}{w_0^{\alpha}} \cdot \left[\frac{(\chi^{\varepsilon} \cdot \mu)^{1-\xi}}{p_0^{\varepsilon}}\right]^{\alpha-1}.$$

The implicit functions defined by these equations have exactly the same properties as the functions F, G, and H. Hence, all the comparative statics of the fixprice equilibrium remain valid in the equilibrium with partially rigid prices.

Appendix C: Optimal Control Problems

This appendix solves the optimal control problems of the household and firm in the dynamic model of Section 4.

Optimal Control Problem of the Household. To solve this problem, we set up the current-value Hamiltonian:

$$\begin{aligned}\mathscr{H}(t,c(t),y(t),m(t)) =& U(c(t),m(t)) + Y(t) \cdot \left[\frac{q(x(t))}{\rho} \cdot (y(t)-c(t)) - s \cdot y(t)\right] \\ &+ M(t) \cdot [p(t) \cdot w(t) \cdot l(t) - p(t) \cdot y(t) - \zeta(t) \cdot m(t) + T(t)]\end{aligned}$$

with control variable $c(t)$, state variables $y(t)$ and $m(t)$, and current-value costate variables $Y(t)$ and $M(t)$. For ease of notation, we have defined

$$U(c(t),m(t)) = \left(\frac{\chi}{1+\chi} \cdot c(t)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+\chi} \cdot m(t)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

The necessary conditions for an interior solution to this maximization problem are $\partial\mathscr{H}/\partial c(t) = 0$, $\partial\mathscr{H}/\partial y(t) = \sigma \cdot Y(t) - \dot{y}(t)$, and $\partial\mathscr{H}/\partial m(t) = \sigma \cdot M(t) - \dot{m}(t)$, together with the transversality conditions $\lim_{t\to+\infty} e^{-\sigma\cdot t} \cdot Y(t) \cdot y(t) = 0$ and $\lim_{t\to+\infty} e^{-\sigma\cdot t} \cdot M(t) \cdot m(t) = 0$. Given that U is concave in (c,m) and that $\mathscr{H}$ is the sum of U and a linear function of (c,y), $\mathscr{H}$ is concave in (c,m,y), and these conditions are also sufficient.

These three conditions can be rewritten as

$$\begin{aligned}
\frac{\partial U}{\partial c(t)} &= Y(t) \cdot \frac{q(x(t))}{\rho} \\
Y(t) \cdot \left(\frac{q(x(t))}{\rho} - s \right) - M(t) \cdot p(t) &= \sigma \cdot Y(t) - \dot{y}(t) \\
\frac{\partial U}{\partial m(t)} &= \zeta \cdot M(t) + \sigma \cdot M(t) - \dot{m}(t)
\end{aligned}$$

In steady state, $\dot{m}(t) = \dot{y}(t) = 0$. Hence, after eliminating the costate variables M and Y, we obtain the following optimality condition:

$$\frac{\partial U}{\partial c} \cdot \left[1 - (\sigma + s) \cdot \frac{\rho}{q(x)} \right] = \frac{p(t)}{\sigma + \zeta} \cdot \frac{\partial U}{\partial m}.$$

Rearranging the terms and computing the derivatives of U, we find that the optimal consumption decision of the household satisfies

$$c = \frac{\chi^{\varepsilon} \cdot (\sigma + \zeta)^{\varepsilon} \cdot m}{p^{\varepsilon}} \cdot \left[1 - (\sigma + s) \cdot \frac{\rho}{q(x)} \right]^{\varepsilon}.$$

Optimal Control Problem of the Firm. To solve this problem, we set up the current-value Hamiltonian:

$$\begin{aligned}
\mathscr{H}(t, n(t), y(t), l(t)) =& y(t) - w(t) \cdot l(t) + Y(t) \cdot [f(x(t)) \cdot (a \cdot n(t)^{\alpha} - y(t)) - s \cdot y(t)] \\
&+ L(t) \cdot \left[\frac{\hat{q}(\theta(t))}{\hat{\rho}} \cdot (l(t) - n(t)) - \hat{s} \cdot l(t) \right]
\end{aligned}$$

with control variable $n(t)$, state variables $y(t)$ and $l(t)$, and current-value costate variables $Y(t)$ and $L(t)$. The necessary conditions for an interior solution to this maximization problem are $\partial \mathscr{H} / \partial n(t) = 0$, $\partial \mathscr{H} / \partial y(t) = \sigma \cdot Y(t) - \dot{y}(t)$, and $\partial \mathscr{H} / \partial l(t) = \sigma \cdot L(t) - \dot{l}(t)$, together with the transversality conditions $\lim_{t \to +\infty} e^{-\sigma \cdot t} \cdot Y(t) \cdot y(t) = 0$ and $\lim_{t \to +\infty} e^{-\sigma \cdot t} \cdot L(t) \cdot l(t) = 0$. Given that $\mathscr{H}$ is concave in (n, l, y), these conditions are also sufficient.

These three conditions can be rewritten as

$$\begin{aligned}
Y(t) \cdot f(x(t)) \cdot \alpha \cdot a \cdot n(t)^{\alpha - 1} &= L(t) \cdot \frac{\hat{q}(\theta(t))}{\hat{\rho}} \\
1 &= Y(t) \cdot (f(x(t)) + s) + \sigma \cdot Y(t) - \dot{y}(t) \\
L(t) \cdot \left(\frac{\hat{q}(\theta(t))}{\hat{\rho}} - \hat{s} \right) &= w(t) + \sigma \cdot L(t) - \dot{l}(t)
\end{aligned}$$

In steady state, $\dot{l}(t) = \dot{y}(t) = 0$. Hence, after eliminating the costate variables L and Y, we find

that the optimal employment decision of the firm satisfies

$$n = \left\{ \frac{\alpha \cdot a}{w} \cdot \frac{f(x)}{\sigma + s + f(x)} \cdot \left[1 - (\sigma + \hat{s}) \cdot \frac{\hat{\rho}}{\hat{q}(\theta)} \right] \right\}^{\frac{1}{1-\alpha}}.$$