Income Volatility and Certification Duration for WIC Children

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Chapter 9 (pp. 259-294) in:
Volatility and Food Assistance in the United States
Dean Jolliffe, James P. Ziliak, eds.
Kalamazoo, MI: W.E. Upjohn Institute for Employment Research, 2008
DOI: 10.17848/9781435684126.ch9

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Income Volatility and Certification Duration for WIC Children

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The U.S. Department of Agriculture administers 15 domestic food assistance programs, including the Special Supplemental Nutrition Program for Women, Infants, and Children (WIC). Policymakers establish program benefits and set criteria that determine who is eligible to receive benefits. WIC benefits include nutrition counseling, health referrals, and vouchers (“food instruments”) that enable WIC clients to obtain particular sets of nutritious foods from authorized retailers. A common criterion for food assistance programs is a limit on income relative to the poverty line. The household income limit for WIC is 185 percent of poverty.1

People’s incomes are not steady forever. Income volatility implies that, in any given month, the household income of a WIC client might exceed the guidelines for eligibility. At a recertification, WIC obtains updated information from the client. Recertification is an administrative tool by which WIC can ascertain whether a client has become ineligible. When a client is detected to be ineligible, WIC benefits are terminated. When a client is found to be eligible, the client may receive WIC benefits until the next recertification. The length of time between recertifications is known as certification duration.

I develop an economic model of an “optimal” certification duration that examines the policy tradeoff between recertification costs and benefit targeting—getting program benefits to those who are eligible to receive them. While the model has implications for food assistance programs in general, I use certifications for WIC children as a case study for simulations of optimal certification durations.

WIC benefits will be called warranted if the client is eligible in the same month that the client receives the program benefits—i.e., if the
client’s current monthly income meets the monthly eligibility guidelines. If instead the client is ineligible in the current month because of income volatility since the application, the client’s benefits will be called *unwarranted*. WIC will terminate the client’s benefits at recertification once benefits are detected to be unwarranted.

A shorter recertification period fosters targeting of benefits to those clients who are eligible. Frequent recertifications detect and terminate unwarranted benefits more quickly. Because WIC benefits constitute a transfer payment from taxpayers to clients, it may seem that terminating payments to ineligibles is simply a zero-sum change. However, because of taxation, financing WIC benefits entails a marginal efficiency cost. Improved benefit targeting that reduces unwarranted benefits provides economic savings in terms of reduced excess burden (deadweight loss) from taxes. Thus there is a social gain to a shorter certification duration.

On the other hand, shortening the certification duration adds to recertification costs. Real resources involved with recertification include staff and equipment costs to WIC (and ultimately, to taxpayers) and a client’s opportunity cost of time and out-of-pocket travel expenses.

The next section provides a static benefit-cost framework to analyze whether or not to conduct a single recertification. This artificially simple framework neglects recertification’s critical intertemporal issues to focus first on valuation issues of what factors constitute economic benefits and costs for the problem. The third section develops the optimal certification duration model, which captures recertification’s dynamic aspects. The fourth section uses the certification duration model to simulate sets of optimal certification duration for children in WIC. The structure of the certification duration model resembles “inspection” or “preparedness” models for maintenance and replacement of stochastically failing equipment, including work by Barlow and Proschan (1996); Jorgenson, McCall, and Radner (1967); and Radner and Jorgenson (1962). The simulations most closely resemble work in Greenfield and Persselin (2002).

The case study involves WIC children. There are five groups of WIC participants: 1) pregnant women, 2) breast-feeding women, 3) non-breast-feeding postpartum women, 4) infants (up to one year old), and 5) children (one through four years of age). Total food costs for WIC in fiscal 2005 were $3.6 billion. The certification duration issue can be
considered more salient for WIC children than for other WIC clients, since children make up half of the program’s participants,^{3} have the longest period for which they can be eligible—up to four years—and are recertified in WIC every six months, a time frame that potentially constitutes several recertifications over a full four-year participation period.

While WIC’s actual certification duration for children is six months, the estimated optimal certification duration is 12 months in a baseline simulation that uses best-guess values for parameters. When the simulation is rerun for sensitivity analysis using alternative parameter values, the estimated optimal certification duration ranges from 7 months to 14 months.

**STATIC BENEFIT-COST ANALYSIS**

At a moment in time, under what conditions does conducting a recertification for a given client pass a benefit-cost test? Suppose the current month is just beginning, and the client has not yet received monthly WIC benefits worth \(M\). While the client was eligible at application, some time has now passed. The client’s eligibility status is unknown unless a recertification is conducted, which is the policy choice. Let \(P_{ei}\) measure the probability that the client, who was eligible at application, is now ineligible. In expectation, unwarranted and warranted benefits equal \(P_{ei}M\) and \((1 - P_{ei})M\), respectively. Let \(c_A\) and \(c_C\) represent recertification costs paid (if and only if a recertification is conducted) by the WIC agency and the client, respectively. The excess burden, or marginal efficiency cost, per dollar of taxes is given by \(\varepsilon\).

Table 9.1 compares a policy of “Don’t recertify” and a policy of “Recertify once” for the current month in isolation, without considering future possible months; the next section takes the future into account. For the policy of “Don’t recertify” there are no recertification costs. Program benefits take on three values: 1) the client receives \(M\) dollars’ worth of monthly WIC benefits (whether or not the client is currently eligible); 2) the taxpayer bears a cost of \(-M\) and, in addition, an excess burden of \(-\varepsilon M\), reflecting taxation’s efficiency loss; and 3) the economic cost to society of WIC benefits, after taking into account the
### Table 9.1 Static Benefit-Cost Analysis of Recertification

<table>
<thead>
<tr>
<th></th>
<th>Client’s benefits and costs</th>
<th>Taxpayer’s benefits and costs</th>
<th>Social benefits and costs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Don’t recertify</strong> policy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Recertification costs</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(2) Program benefits</td>
<td>+ $M$</td>
<td>-(1 + $\varepsilon$)$M$</td>
<td>-$\varepsilon$M</td>
</tr>
<tr>
<td>(3) $E {\text{net value}}$</td>
<td>(3) = (1)+(2)</td>
<td></td>
<td>-$\varepsilon$M</td>
</tr>
<tr>
<td><strong>Recertify once</strong> policy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) Recertification costs</td>
<td>-$c_C$</td>
<td>-(1 + $\varepsilon$)$c_A$</td>
<td>-[(1 + $\varepsilon$) $c_A$ + $c_C$]</td>
</tr>
<tr>
<td>(5) Program benefits if ineligible (probability $P_{EI}$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(6) Program benefits if eligible (probability [1 - $P_{EI}$])</td>
<td>+ $M$</td>
<td>-(1 + $\varepsilon$)$M$</td>
<td>-$\varepsilon$M</td>
</tr>
<tr>
<td>(7) $E {\text{net value}}$</td>
<td>(7) = (4) + [$P_{EI}(5) + (1 - P_{EI})(6)$]</td>
<td></td>
<td>(1 - $P_{EI}$)$\varepsilon$M - [(1 + $\varepsilon$) $c_A$ + $c_C$]</td>
</tr>
</tbody>
</table>

**Benefit-cost test of “recertify once” policy vs. “don’t recertify” policy**

(8) $E \{\text{net value}\}$

(8) = (7) - (3)

+ $P_{EI}$($\varepsilon$)$M$ - [(1 + $\varepsilon$) $c_A$ + $c_C$]

**SOURCE:** Author’s analysis.
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transfer-payment aspect of $M$, is $-\varepsilon M$ in row (3). Rows (4) through (7) examine the “Recertify once” policy. Recertification costs are shown in (4). In (5), if with probability $P_{EI}$ the client is currently ineligible, WIC benefits are terminated. In (6), with probability $(1 - P_{EI})$ the client is currently eligible, and WIC benefits are paid. The expected net value in row (7) of the “Recertify once” policy shows terms that are the total of (4) and the probability-weighted values in (5) and (6).

Row (8) shows the net gain of adopting the “Recertify once” policy in place of the “Don’t recertify” policy as the difference between (7) and (3):

\[(9.1) \quad E[NV] = P_{EI}\varepsilon M - [(1 + \varepsilon)c_A + c_C].\]

Recertification passes a static benefit-cost test when the $E[NV]$ in Equation (9.1) is positive. In Equation (9.1), recertification saves neither warranted benefits nor their excess burden because the policy choice does not affect payments of warranted benefits. Under the “Recertify once” policy, WIC benefits are paid out so long as they are warranted, and warranted benefits (like unwarranted benefits) are also paid under the “Don’t recertify” policy. Instead, (9.1) shows that the economic gain from recertification is $P_{EI}\varepsilon M$—the excess burden of financing WIC benefits, $\varepsilon M$, with probability $P_{EI}$ that those benefits are unwarranted. The link to unwarranted benefits can also be shown by rewriting that gain as $\varepsilon(P_{EI}M)$, where unwarranted benefits are $P_{EI}M$. Because the term $\varepsilon M$ recurs often in the optimal certification duration model, it will simply be designated $m$.

In the static benefit-cost analysis, a “Recertify once” policy tends to have a positive $E[NV]$ when, holding other factors constant, any one of the following four conditions occur: 1) “income volatility” is relatively high (in the sense of a high value for $P_{EI}$), 2) monthly benefits $M$ are relatively high, 3) the excess burden of taxation $\varepsilon$ is relatively high, or 4) the recertification costs for the agency and the client are relatively low. These same lessons can be expected to hold more generally for other food assistance programs.
The static benefit-cost analysis highlighted valuation issues, but it ignored intertemporal issues that matter for recertification policy. First, the static analysis treated $P_{EI}$ as an exogenous constant, but $P_{EI}(t)$ can systematically depend on time. Second, if ineligibility is detected and benefits are terminated at some time, excess burden $m$ is saved not only for the current month but also for future months that would have had payments if WIC benefits had continued until a future recertification. Thus, future flows of $m$ and their expected discounted values will matter for optimal decision-making. Third, the static analysis did not model how decision-making for any one recertification affects outcomes of possible future recertifications. The optimal certification duration model takes into account these three issues, and this section considers each in turn.

**Income Volatility and State Probability Paths**

At the time of application and initial certification, time 0, the WIC agency determines that the client is eligible. The time of transition from eligibility to ineligibility is a random variable that can be called a time to “failure.” The failure distribution $F(t)$ is the cumulative probability distribution showing the probability that the time to “failure”—ineligibility—is less than or equal to $t$.

$$F(t) = \int_0^t f(u)du = \text{Prob}\{\text{time to failure} \leq t\}$$

where $f(t)$ is the associated probability density for $F(t)$ given by $dF(t)/dt$. Although the client’s current state—eligible or ineligible—is unknown, it is assumed that $F(t)$ is known to policymakers from past data on other clients that resemble the one being considered. A companion to the failure distribution is the reliability (or survivor) function $R(t)$, which is the probability a client is still eligible as of time $t$—or, equivalently, the probability that the time of transition to ineligibility is greater than $t$. The relationship between $R(t)$ and $F(t)$ is simply

$$R(t) = 1 - F(t) = \text{Prob}\{\text{time to failure} > t\}.$$
The hazard rate (also known as the “hazard function” or the “failure rate”) \( h(t) \) is the probability that the client will transition to ineligibility within an instantaneously small interval at time \( t \), conditioned on having reached \( t \) as eligible (i.e., conditioned on not transitioning prior to \( t \)). The hazard rate can be expressed using the failure distribution or the reliability function:

\[
(9.4) \quad h(t) = \frac{F'(t)}{1 - F(t)} = \frac{R'(t)}{R(t)}.
\]

Like previous work, the certification duration model adopts the common simplifying assumption that the reliability function for the eligible client to remain eligible is exponential:

\[
(9.5) \quad R(t) = e^{-\lambda t}, \quad t \geq 0.
\]

The hazard rate of (9.5) for transition from \( E \) to \( I \) is a constant \( \lambda \). The failure distribution is

\[
(9.6) \quad F(t) = 1 - e^{-\lambda t}, \quad t \geq 0.
\]

The expected duration of a continuous spell in \( E \) is \((1/\lambda)\) from the start of the spell.

The exponential distribution has been used in equipment maintenance and inspection models by Barlow and Proschan (1996); Jorgenson, McCall, and Radner (1967); and Radner and Jorgenson (1962). A key feature of the exponential distribution is its “memoryless” property, by which the reliability function for equipment that is inspected and found to be in good working order at some time is the same reliability function for the newly installed equipment as of time 0. The hazard rate to failure for a newly installed piece of equipment is \( \lambda \), and at any point in the future—before or after inspection—the hazard rate is still \( \lambda \). Inspection serves as a “regeneration” or “renewal” that returns the problem’s stochastic characteristics to time 0.4

All models make simplifying assumptions. Despite the potential limitations of the exponential function, its advantage is that it provides the easiest dynamic structure for the optimal certification duration model. Moreover, in the next section, data are presented that show that the actual probability paths for some households’ income dynamics
are approximated well by the theoretical probability path predicted on the basis of the exponential distribution. Even if the assumption of an exponential distribution does not hold precisely in actual data, much can be learned about the trade-offs faced by program administrators by considering the optimal certification duration model. The more closely actual hazard rates are constants, the better the model’s simulations will serve as first-order approximations.

A constant hazard rate is a strength of the exponential distribution (making difficult problems analytically tractable) as well as a potential limitation. More advanced models that allow for nonconstant hazard rates may build on the foundations provided by the optimal certification duration model developed here. When hazard rates depend on time, the model’s solution would almost certainly involve a set of sequential recertification periods of differing lengths. A second limitation of the exponential distribution, and one shared by many distributions, is that its cumulative probability of transition approaches 1.0. It would be of interest to generalize the model by allowing some fraction of WIC clients to have zero probability of “failure,” i.e., of ever having enough income to become ineligible. That complexity is not attempted here.

A complexity that is incorporated into the certification duration model, generalizing the models of the equipment maintenance literature, is permitting transitions not only from I to E but also from E to I. It is natural for analysis of optimal equipment maintenance to adopt the assumption that a transition into “failed” is irreversible: machines do not spontaneously fix themselves. The optimal certification duration model relaxes the assumption that failure is an “absorbing” state, with no return possible, because income volatility can move a household from eligibility to ineligibility and, at some random time later, back into eligibility. Such a possibility is captured by a “two-state transition process” or “alternating renewal process.” Boskin and Nold (1975) considered it when examining the states of “participation” and “nonparticipation” in modeling the AFDC program. Lancaster (1990) considered it for the states of employment and unemployment.

In the next section, titled “Application: WIC Children’s Certification Duration,” data are presented that show that the actual two-state probability paths for some households’ income dynamics are approximated well by the theoretical probability path predicted on the basis of the exponential distribution.
Let $G(t)$ represent the failure distribution for transition from $I$ to $E$. $G(t)$ pertains to a formerly eligible, currently ineligible client who reenters $E$ because of an income decrease, where

\begin{equation}
G(t) = 1 - e^{-\mu t}, \quad t > 0.
\end{equation}

A (formerly eligible) client who is ineligible at any given time exhibits the hazard rate $\mu$ for the conditional probability of transitioning from $I$ back to $E$.

The expected duration of a continuous spell in $I$ is $(1/\mu)$ from the start of the spell. Just as inspection returned stochastic properties back to time 0 in equipment maintenance models that used a one-way transition assumption, so too does recertification serve as a regeneration point under two-way transitions between $E$ and $I$. Let the probability that a client occupies a given state $E$ or $I$ at a given time $t$ be a “state probability,” given by $\pi_E(t)$ and $\pi_I(t)$, respectively. Generalizing a numerical example from Howard (1960), state probabilities are either

\begin{equation}
\pi_E(t) = \pi_E(0) \left[ \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} \right] + \pi_I(0) \left[ \frac{\mu}{\lambda + \mu} - \frac{\mu}{\lambda + \mu} e^{-(\lambda + \mu)t} \right]
\end{equation}

or

\begin{equation}
\pi_I(t) = \pi_E(0) \left[ \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} \right] + \pi_I(0) \left[ \frac{\lambda}{\lambda + \mu} + \frac{\mu}{\lambda + \mu} e^{-(\lambda + \mu)t} \right],
\end{equation}

where the probabilities $\pi_E(0)$ and $\pi_I(0)$ represent the probability that the client is in $E$ or $I$ at whatever “initial” time 0 is considered. Suppose the initial time of the problem is taken to be the time of application (and initial certification) of the client. At application, the WIC agency knows that $\pi_E(0) = 1$ and $\pi_I(0) = 0$. For the eligible client, (9.8) and (9.9) each reduce to a set of conditional state probabilities, i.e., to state probabilities (for $E$ or $I$) conditioned on initial eligibility at time 0, which can be written as $P_{EE}(t)$ and $P_{EI}(t)$:

\begin{equation}
P_{EE}(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t};
\end{equation}
Not surprisingly, the stochastic time profiles $P_{EE}(t)$ and $P_{EI}(t)$ sum to 1 at all $t$: a client (who is eligible at time zero) is either in state $E$ or state $I$ at any given time. These state probability paths approach steady-state values monotonically: $P_{EE}(t)$ is downward-sloping for all $t$, while $P_{EI}(t)$ is upward-sloping. What those behaviors mean, in practice, is that the probability of detecting that a client is ineligible at time $t$ strictly increases as more time passes.

Consider next a recertification after some amount of time $T$ has passed since application or the last recertification. Because of the memoryless property, the recertification serves as a “regeneration” of the two-state stochastic process of transitioning between states $E$ and $I$. The stochastic time profiles $P_{EE}(t)$ and $P_{EI}(t)$ hold for an eligible client at time 0 by using $\pi_E(0) = 1$ and $\pi_I(0) = 0$ in (9.8) and (9.9). Similarly, at recertification at $T$, $\pi_E(T) = 1$ and $\pi_I(T) = 0$ for the client who is found, because of the process of recertification itself, to be eligible at $T$. Using those values in (9.8) and (9.9) again results in the same stochastic profiles at $T$ as were found at time 0. That is, viewed from time $T$, the same $P_{EE}(t)$ and $P_{EI}(t)$ time paths are obtained that were derived at initial time 0 (so long as $t$ is interpreted as “duration time from the last recertification” rather than calendar time). In short, recertification can serve as a new initial time. The same result holds at each successive recertification—at $2T$, $3T$, and so forth after time 0.

A Single Optimal Recertification

The central role of discounting in a dynamic certification duration model is considered in the context of the optimal timing of a single recertification at time $T$, to be chosen by WIC. The per-certification cost is given by $c$ equaling $[(1 + \varepsilon) c_A + c_C]$, as in the static benefit-cost test. The present value of $c$ at time 0, with $\rho$ as the instantaneous social discount rate, is $D(T)$:

(9.12) \[ D(T) = e^{-\rho T} c. \]

A delay in recertification lowers $D(T)$ through postponement of the recertification cost $c$. 

(9.11) \[ P_{EI}(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu) t}. \]
Let $U(T)$ measure the expected discounted excess burden of unwarranted benefits that cumulate up to recertification time $T$:

\[
U(T) = m \int_0^T e^{-\rho t} P_{EI}(t) \, dt = m \int_0^T e^{-\rho t} \left[ \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu) t} \right] \, dt.
\]

$U(T)$ has three components. The excess burden component $m$ (where $m$ is $\varepsilon M$) measures the flow of potential savings of excess burden from financing WIC benefits. The state probability path $P_{EI}(t)$ registers for each instant the likelihood that the excess burden from unwarranted benefits occurs at that moment. Each instant’s flow is discounted by $\rho$ to initial time. The cumulative aspect of $U(T)$ means that these expected discounted flows are added up by integration between 0 and $T$.

Upon evaluation of (4.2), $U(T)$ is given by

\[
U(T) = \left( \frac{\lambda}{\lambda + \mu} \right) m \left[ \frac{1}{\rho} \left(1 - e^{-\rho T}\right) - \frac{1}{\rho + \lambda + \mu} \left(1 - e^{-(\rho + \lambda + \mu) T}\right) \right].
\]

The present value of an infinite stream of $m$ is $m/\rho$. Importantly, the value of $U(T)$ in the limit takes on a smaller value, as follows:

\[
U_\infty = \lim_{T \to \infty} U(T) = \left( \frac{\lambda}{\rho + \lambda + \mu} \right) \left( \frac{m}{\rho} \right).
\]

Thus, if no inspection were ever to occur—i.e., if $T$ were infinitely delayed—the expected excess burden of unwarranted benefits would be less than $m/\rho$ because (in expectation) only a portion of the flow of excess burden $m$ is paid to a client during times of ineligibility. The value that $U(T)$ approaches will be closer to $(m/\rho)$ as $\lambda$ is larger and $\mu$ is smaller relative to $\rho$, inasmuch as those income volatility parameters capture the rates at which clients have $E$-$I$ and $I$-$E$ transitions.

Expected discounted total cost for a single-recertification problem is given by

\[
E[DC(T)] = U(T) + D(T) + e^{-\rho T} \left[ P_{EI}(T) 0 + P_{EI}(T) U_\infty \right].
\]

In (9.16), a flow of excess burden from unwarranted benefits is accruing in expectation until time $T$, given by $U(T)$. At $T$, a recertification
cost is paid (with certainty) for which the present value is \(D(T)\). At recertification the client is found to be ineligible with probability \(P_{EI}(T)\), in which case payments are terminated (meaning future payments equal 0) and the problem is over. Or, with probability \(P_{EE}(T)\), the client is found to be eligible and the payment of program benefits forever begins. That is, if the client passes the problem’s single recertification, the flow of \(m\) continues uninterrupted from \(T\) onward without limit. Discounting the two probability-weighted outcomes of recertification at \(T\) (of 0 and of \(U_\infty\)) back to initial time 0, in order to express them in present value terms, completes (9.16).

Minimization of (9.16) results in the first-order condition

\[
(9.17) \quad P_{EI}(T^*)m - \rho c + P_{EE}(T^*)U_\infty - \rho P_{EE}(T^*)U_\infty = 0 .
\]

The problem’s second-order condition for minimization is

\[
(9.18) \quad P_{EI}'(T^*)m + P_{EE}''(T^*)U_\infty - \rho P_{EE}'(T^*)U_\infty > 0 .
\]

The optimality condition (9.17) can be reexpressed in terms of marginal gains and marginal losses from delayed recertification. To use a single state probability path and its derivative, note that \(P_{EI}'(t) = -P_{EE}'(t)\), which follows from \(P_{EI}(t) = [1 - P_{EE}(t)]\), so that (9.17) becomes

\[
(9.19) \quad P_{EI}'(T^*)U_\infty + \rho c + \rho[1 - P_{EI}(T^*)]U_\infty = P_{EI}(T^*)m .
\]

The first term shows a gain from delayed recertification: a slight delay in \(T^*\) increases the probability that ineligibility will be detected at the recertification, thereby saving \(U_\infty\). The second term captures an additional gain from delaying payment of the per-recertification cost \(c\): that cost will be paid sooner or later, but if it is later then in effect the implicit interest earnings from delayed payment equal \(\rho c\). That is, if \(c\) were invested in the bank, earning instantaneous rate \(\rho\) until recertification, then proceeds from waiting until the next instant for recertification and earning \(\rho\) meanwhile on the invested capital of \(c\) is \(\rho c\). There is another gain associated with interest earnings. As of \(T\), there is a probability \([1 - P_{EI}(T)]\) that the client is eligible and that at recertification the client will be due monthly benefits \(M\) forever, costing \(m\) forever, which is tantamount to a lump-sum cost (in present value as of \(T\) of
A slight delay in recertification means that that lump-sum value is not paid yet. The lump-sum value can be invested in the bank to accrue interest, resulting in the third term. The term on the right-hand side is the instantaneous cost of delayed recertification. That cost results from not terminating the stream of \(m\) for a client who is already ineligible (with probability \(P_{E|}[T]\)). The optimal \(T^*\) in (9.19) balances these four terms.

Making use of (9.15) to express \(U_\infty\) in (9.19) as a function of \(m\) (and the parameters \(\lambda, \mu, \) and \(\rho\)) and then dividing (9.19) throughout by \(m\) reveals an important feature of the problem. At the optimum, the only dependence \(T^*\) has on \(c\) and on \(m\) is through the term \((\rho c/m)\)—i.e., through their relative values.

**Optimal Periodic Recertifications**

The single-recertification problem just considered resembles the tree-cutting problem of Fisher. Both problems assume that an action—whether recertification or tree harvesting—is taken once, whereupon the problem is over. In this section, the optimal certification duration model incorporates multiple stochastic cycles to recognize that a decision about any one recertification affects the likelihoods and outcomes of future recertifications.

Suppose WIC determined at time 0 that the optimal length of the first certification duration is \(T_1^*\). For a client who is eligible at the first recertification, the problem as of \(T_1^*\) for the second certification duration, \(T_2^*\), resembles precisely, in its stochastic specification, the problem as of time 0 for how long to specify the first certification duration (because of the memoryless property and the infinite-horizon feature of the model). Thus, the problem will yield the same solution, that \(T_2^* = T_1^*\). Moreover, each certification duration will be the same optimal length. Although there are multiple cycles, the problem condenses to selecting a single optimal \(T^*\), taking into account all possible future cycles. The problem will have recertifications at times \(T^*, 2T^*, 3T^*, \) and further multiples of \(T^*\).

\(S(T)\) and \(M(T)\) refer to costs associated with a single cycle and with multiple cycles, respectively. \(M(T)\) is derived from \(S(T)\). The expected discounted cost of the (first) cycle from 0 to \(T\) is given by

\[
E[S(T)] = U(T) + D(T)
\]
The within-cycle discounted cost of the second cycle from $T$ to $2T$, or within any future cycle, is also $E[S(T)]$. Because $E[S(T)]$ appropriately discounts within-cycle costs only to the start of the cycle, the present value of a cycle’s cost involves discounting that value back to time 0. Conditioned on the client being eligible at $T$, that value for the second cycle would be

$$ PV\{\text{Cycle #2 | eligibility at } T\} = e^{-\rho T} E[S(T)], $$

and more generally for future cycles $n = 2, 3, 4, \ldots$, the value is

$$ PV\{\text{Cycle #} n | \text{eligibility at } (n-1)T\} = e^{-\rho (n-1)T} E[S(T)]. $$

In (9.22) the present value is conditioned on the client being eligible at a given recertification. As of time 0, those outcomes are uncertain. For example, only with a probability $P_{EE}(T)$ will the client be found to be eligible at the first recertification. With probability $P_{EI}(T)$ the client is ineligible and benefits are terminated. The second cycle’s (unconditioned) expected present value is

$$ EPV\{\text{Cycle #2}\} = e^{-\rho T} \left[ P_{EE}(T) E[S(T)] + P_{ EI}(T)(0) \right]. $$

The third cycle commences if the client passes the second recertification, and that is possible only if the client has already passed the first recertification with probability $P_{EE}(T)$:

$$ EPV\{\text{Cycle #3}\} = P_{EE}(T) e^{-\rho T} \left[ P_{EE}(T) E[S(T)] + P_{ EI}(T)(0) \right]. $$

The fourth cycle is reached on condition that the third cycle was begun, an event that requires passing two recertifications in a row with probability $P_{EE}(T)^2$. Most generally,

$$ EPV\{\text{Cycle #} n\} = \left[ P_{EE}(T) \right]^{n-2} e^{-\rho (n-1)T} \left[ P_{EE}(T) E[S(T)] + P_{ EI}(T)(0) \right], $$

where the term $\left[ P_{EE}(T) \right]^{n-2}$ reflects the number of recertifications that must be passed with uninterrupted successes to reach the start of any given cycle $n$.9
$M(T)$ is the sum of the expected present value of each successive cycle:

\begin{equation}
M(T) = E[S(T)] + e^{-\rho T} \left[ P_{EE}(T) E[S(T)] + P_{EI}(T)(0) \right] \\
+ P_{EE}(T)e^{-\rho T} \left[ P_{EE}(T) E[S(T)] + P_{EI}(T)(0) \right] \\
+ [P_{EE}(T)]^2 e^{-\rho T} \left[ P_{EE}(T) E[S(T)] + P_{EI}(T)(0) \right] + \cdots \\
= E[S(T)] + [e^{-\rho T} P_{EE}(T)] E[S(T)] + [e^{-\rho T} P_{EE}(T)]^2 E[S(T)] \\
+ [e^{-\rho T} P_{EE}(T)]^3 E[S(T)] + \cdots \\
= \frac{E[S(T)]}{1 - e^{-\rho T} P_{EE}(T)}.
\end{equation}

The final line in (9.26) follows from viewing $M(T)$ as the infinite sum of terms that involve a geometric sequence. It is helpful to express $E[S(T)]$ by its two components, $U(T)$ and $D(T)$:

\begin{equation}
M(T) = \frac{U(T)}{1 - e^{-\rho T} P_{EE}(T)} + \frac{D(T)}{1 - e^{-\rho T} P_{EE}(T)}.
\end{equation}

In (9.27) $M(T)$ is clearly the sum of two cost curves (in expected present value, across multiple stochastic cycles), one for excess burden of unwarranted benefits and one for recertification costs. These two curves are depicted in Figure 9.1, together with the “Total cost” curve given by $M(T)$. Optimal $T^*$ is that value of $T$ at which $M(T)$ is minimized.

In the simulation, $M(T)$ is calculated using the functions and parameters in (9.27), and then letting $T$ increase in one-month increments. (An upper limit of 99 months was examined.) The optimal certification duration, at $T^*$, is identified as the last month in which the change $M(T^*) - M(T^* - 1)$ is negative: a month past $T^*$ results in an increase in $M(T)$, so $M(T)$ is minimized at $T^*$.

**Time Horizons**

An infinite-horizon specification embodied in (9.16) and (9.26) is helpful for distilling the dynamic essence of the benefits and costs accrued from delaying recertification. It is appropriate to consider how
well an infinite-horizon specification can be used for conducting simulations of WIC children’s optimal certification duration in the next section.

While in many cases it is less accurate to use an infinite-horizon rather than a finite-horizon specification, the infinite-horizon version is typically less complex. Its degree of accuracy depends, in part, on the length of the appropriate finite horizon and the social discount rate. The longer the finite horizon, the more accurate the infinite-horizon simplification will be. Accuracy is also better when the social discount rate is higher, which decreases the present value of any given future benefit or cost (values that are retained in the infinite-horizon specification but that might be cut off after a finite horizon).
The issue of an appropriate time horizon is pertinent for the case study on WIC children because the length of their participation is at most four years (up to age five). In the end, whether or not an infinite-horizon specification is a good approximation for WIC children depends in part on the numerical results of the simulation. If the so-called optimal certification duration $T^*$ from the simulation is estimated at, say, seven or eight years, the use of the infinite-horizon clearly has a serious weakness: by neglecting the four-year maximum, the estimate $T^*$ exceeds the upper limit of age-based eligibility. On the other hand, if the simulation’s estimated $T^*$ is, say, a few months, the estimate is short relative to a four-year maximum participation, and the difference between infinite- and finite-horizon specifications may be negligible. Even then, however, the simulation’s optimal certification duration does fit best for a one-year-old, who has the longest potential participation. Accuracy diminishes when a child is older and thus closer to the upper limit on age requirements.

**APPLICATION: WIC CHILDREN’S CERTIFICATION DURATION**

This section uses the optimal certification duration model to simulate $T^*$ for a case study of WIC children. Conducting any simulation exercise involves more than just specifying an economic model and assigning values for key parameters. While there has been some research done on certification activities for local WIC agencies (e.g., Macro International 1995) certification costs capturing the time involved, from both a WIC agency’s and a client’s perspective, have not been studied as much for WIC as for food stamps. And while some work has been carried out on dynamic income patterns and labor market behavior for families with WIC infants and children, the simulations need greater month-by-month detail than previous work has presented. Information on the dollar cost of the WIC food package for children is incomplete. Of necessity, the simulations rest on assumptions used to develop a set of values for model parameters. A baseline simulation is conducted using best-guess values of parameters. To ascertain the effects of alternative assumptions, additional simulations are done as a sensitivity
analysis using alternative parameter values. The simulations for optional certification durations are tentative, subject to refinement by improving the model or the estimates of parameters.

**State Probability Paths of Households with Older Children**

A longitudinal approach is needed that will allow the income trajectories of individual households to be followed over time in order to identify which months a household is eligible and which months a household is ineligible. The simulations draw on results by Newman (2006), who identified a set of households that were income-eligible for free or reduced-price meals in the National School Lunch Program (NSLP) at a particular moment (August at the start of the school year). Newman followed that set of households over time to examine, for the set as a whole, what percentage were not income-eligible in each successive month, yielding a state probability path $P_{t\omega}(t)$ for NSLP. There is a basis for using Newman’s NSLP results for a simulation on WIC children: the income limit for reduced-price meals is the same—185 percent of poverty—as WIC’s income limit. The income dynamics for households with older school-aged children are used to approximate the income dynamics for households with younger children of WIC age.11

It so happens that the Newman study contained three moments of eligibility, each for the month of August, but in three successive years, 1996 through 1998. The first three columns of Table 9.2 show Newman’s results by year; the data make use of the seam-adjusted share figures reported in Newman (2006).

The table shows that in month 0 (August of each respective year), every household retained in the subsample is eligible, making 0.0 percent ineligible. Then, as the months of the school year pass, the percentage of households that are ineligible in a given month increases at a decreasing rate. The table also shows, in the fourth column, the monthly averages across the three years’ state probability figures, which yields an average state probability path. The average path approaches a value that is taken to be a long-run or “steady state” value. The transitional nature of the average probability path is seen more readily in Figure 9.2, which depicts graphically the table’s three-year average path (and a fitted path, described below).12 In order to separate and estimate implicit exponential exit and reentry rates for eligibility, the steady-state
probability $\mu/(\lambda+\mu)$ toward which the state probability path approaches was set (by visual inspection) at 0.25. The rate of approach towards the steady-state probability was used to estimate the state probability path using OLS. From Equation 9.11, it follows that

$$\ln \left( 1 - \frac{P_{\mu}(t)}{0.25} \right) = - (\lambda + \mu) t ,$$

where the average figures of column 4 are used for the data on $P_{\mu}(t)$ in (9.28). The coefficient on time was estimated to be $-0.309$ (with the restriction of zero-intercept imposed), with an $R$-squared value of 0.85 for the equation. Based on the estimated coefficient, the estimated (monthly) value of $\lambda$ was 7.725 percent, and $\mu$ was estimated at 23.175 percent. It is no accident that the estimated value of $\mu$ is precisely three times the estimated value of $\lambda$: that relationship is implied by the stipulation that the steady-state probability $\mu/(\lambda+\mu) = 0.25$, and it serves as the identification restriction by which the two structural parameters are obtained from the estimated coefficient on time. The fifth column of the table and its accompanying figure each show a “fitted” state probability path, based on the estimates of the two structural parameters.

Table 9.2  State Probability Paths of Income Ineligibility, 1996–1998
(185 percent of poverty, households with school-aged children)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1</td>
<td>13.7</td>
<td>11.8</td>
<td>7.2</td>
<td>10.9</td>
<td>6.6</td>
</tr>
<tr>
<td>2</td>
<td>12.9</td>
<td>13.1</td>
<td>11.1</td>
<td>12.4</td>
<td>11.5</td>
</tr>
<tr>
<td>3</td>
<td>17.6</td>
<td>21.1</td>
<td>17.0</td>
<td>18.6</td>
<td>15.1</td>
</tr>
<tr>
<td>4</td>
<td>19.5</td>
<td>18.4</td>
<td>16.8</td>
<td>18.2</td>
<td>17.7</td>
</tr>
<tr>
<td>5</td>
<td>20.3</td>
<td>19.8</td>
<td>19.0</td>
<td>19.7</td>
<td>19.7</td>
</tr>
<tr>
<td>6</td>
<td>22.1</td>
<td>23.8</td>
<td>20.0</td>
<td>22.0</td>
<td>21.1</td>
</tr>
<tr>
<td>7</td>
<td>23.0</td>
<td>21.6</td>
<td>18.8</td>
<td>21.1</td>
<td>22.1</td>
</tr>
<tr>
<td>8</td>
<td>22.2</td>
<td>22.0</td>
<td>20.6</td>
<td>21.6</td>
<td>22.9</td>
</tr>
<tr>
<td>9</td>
<td>23.1</td>
<td>23.5</td>
<td>23.8</td>
<td>23.5</td>
<td>23.5</td>
</tr>
<tr>
<td>10</td>
<td>25.9</td>
<td>25.3</td>
<td>22.5</td>
<td>24.6</td>
<td>23.9</td>
</tr>
<tr>
<td>11</td>
<td>23.3</td>
<td>23.2</td>
<td>23.6</td>
<td>23.4</td>
<td>24.2</td>
</tr>
</tbody>
</table>

An $R^2$ value of 0.85 from (9.28) is considered here to be a good fit with the data. It is the empirical support for the claim (in the third section) that the actual probability paths for some households’ income dynamics are approximated well by the theoretical probability path predicted on the basis of the exponential distribution.

**Dollar Cost of WIC Children’s Benefits**

The simulations do not account for the dollar cost of WIC’s nutrition counseling and health referrals: the cost of the WIC food package alone is used.

The simulation of the certification duration model requires a figure for monthly WIC benefits for children. The children’s package con-
tains such items as cereal, peanut butter, juice, cheese, milk, and eggs; participants have some choice regarding combinations of certain items. The cost of a WIC food package for a particular participant category is not routinely collected or estimated. The most comprehensive work in this area, by Davis and Leibtag (2005), examines the role of food prices, caseload composition, and cost-containment practices in affecting a state’s WIC food package costs for 17 selected states under study. For purposes of defining a common cross-state food package standard by which to compare food package costs, Davis and Leibtag used the maximum quantity of food available in each food package. However, while WIC can provide a particular client with a prescription of foods up to the maximum set by federal regulation, WIC tries to tailor the amounts of a package’s individual food items to the individual client to match preferences and avoid waste. The average prescriptions are typically below the maximum prescriptions. For example, in April 2004 the federal maximum prescription for milk was 24 quarts, but only 2.1 percent of children received the federal maximum. At the time, the average quantity of milk prescribed was 16.8 quarts (USDA 2006c). To better approximate the pattern of actual prescriptions, this chapter takes into account, by item, that the average quantity received by a child nationally may be below the federal maximum, resulting in a slight adjustment to the work by Davis and Leibtag. Table 9.3 shows the estimated value of the children’s food package for each of the 17 selected states, based on prices reported by Information Resources Inc. for the market areas in the state; as Davis and Leibtag note, these prices may not be representative of prices in the entire state. The table shows $32.52 as the simple average across the 17 states. This figure is used as the baseline value for M.

**WIC Recertification Costs**

An estimate of agency recertification costs is derived using three factors: a figure for the hourly wage rate for direct staff time, a figure for the direct staff time (in hours) associated with recertification, and a percentage figure for an overhead rate. The cost figure that is derived is for 1998; that year is within the 1997–1999 range for which the WIC food package costs were derived and also within the 1996–1998 range of the Newman income volatility data.
Among the public health nutrition workforce in Full Time Equivalent positions, four-fifths (81 percent) of employees are employed in WIC programs (USDA 2003). Across the 2,200 local agencies and 10,000 WIC service sites, caseload and staffing vary. Some local agencies have one or two staff while others have more than 350 (USDA 2006a, p. 14) Various staff have administrative, nutrition counseling, and medical skills. Tasks involved with certification (as opposed to providing WIC benefits in the form of nutritional counseling or breastfeeding promotion) include determining identity, state of residency, and income eligibility; measuring height and weight; drawing blood and doing analysis; and determining nutritional risk. At the local level, WIC uses a variety of approaches and different combinations of professionals, paraprofessionals, and WIC’s “competent professional authorities” to conduct the various certification tasks. The simulations use Bureau of Labor Statistics (BLS) data to estimate a 1998 figure for total hourly

### Table 9.3 Estimated WIC Children’s Food Package Cost by State, 1997–1999

<table>
<thead>
<tr>
<th>State</th>
<th>Cost of WIC children’s food package ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>34.71</td>
</tr>
<tr>
<td>CO</td>
<td>33.52</td>
</tr>
<tr>
<td>FL</td>
<td>33.15</td>
</tr>
<tr>
<td>GA</td>
<td>31.12</td>
</tr>
<tr>
<td>IA</td>
<td>30.36</td>
</tr>
<tr>
<td>IL</td>
<td>32.79</td>
</tr>
<tr>
<td>KS</td>
<td>32.64</td>
</tr>
<tr>
<td>MA</td>
<td>34.27</td>
</tr>
<tr>
<td>MI</td>
<td>31.60</td>
</tr>
<tr>
<td>MN</td>
<td>30.74</td>
</tr>
<tr>
<td>MO</td>
<td>31.25</td>
</tr>
<tr>
<td>NY</td>
<td>36.27</td>
</tr>
<tr>
<td>PA</td>
<td>32.18</td>
</tr>
<tr>
<td>TN</td>
<td>32.78</td>
</tr>
<tr>
<td>TX</td>
<td>31.24</td>
</tr>
<tr>
<td>WA</td>
<td>34.24</td>
</tr>
<tr>
<td>WI</td>
<td>30.04</td>
</tr>
<tr>
<td>Average</td>
<td>32.52</td>
</tr>
</tbody>
</table>

**SOURCE:** Davis and Leibtag (2005) and author’s analysis.
compensation across “health care and social assistance employees” in state and local government to proxy what weighted-average combination of employees’ wages and times may affect certification cost in any particular locality.\textsuperscript{14}

The National Compensation Survey of the BLS reports that the cost per hour worked (in terms of wages and salaries) for health care and social assistance employees in state and local government averaged $23.53 in the second quarter of 2006, representing 65.9 percent of total compensation (BLS 2008).\textsuperscript{15} Because the BLS recently shifted from the Standard Industrial Classification (SIC) system to the North American Industry Classification System (NAICS), the time series for NAICS figures begins only in 2004; thus, an NAICS figure for 1998 is not available. However, the SIC figure from BLS for 1998 for wages and salaries of “all workers” in state and local government is $19.19 per hour, representing 70.3 percent of total compensation. Thus, the total compensation in 1998 for all workers was $27.30. An adjustment of this figure downward, to better estimate total compensation of health care and social assistance employees who staff WIC, results in an estimate of $25.40 in total compensation per hour for 1998.\textsuperscript{16} This figure is used as an approximation of a weighted-average wage rate across staff involved with WIC certifications.

Among the documents the Office of Management and Budget (OMB) provides for guidance and reference is a general guide to benefit-cost analysis that states the following:

When calculating labor costs, the OMB recommends using prevailing wage rates and salaries. To arrive at fully burdened costs when estimating personnel costs for government employees, you must add overhead costs to salary and fringe-benefit costs. . . . Some examples of indirect costs include rent, utilities, insurance, indirect labor, and other expenses typically charged to the organization as a whole. . . . For evaluation purposes, costs (both direct and indirect) should be included if they will change with the introduction of a proposed system (Federal CIO Council 1999, pp. 14–15).

It is sensible to account for indirect costs. A change in certification policy can have long-term and large-scale effects on the caseload and on the agency’s overall staffing and capacity to service that caseload. If certification periods were changed nationally, states could change (at least in the long term) not only staffing but also office space, equipment,
and the like, to support the new flow of recertifications. Therefore, it is suitable for the certification duration model to use a “burdened” or a “loaded” hourly labor cost rather than considering wage costs alone. The total compensation figure estimated for health care and social assistance employees in state and local government for 1998 was estimated to be $25.40. To cover all indirect costs as well, this paper applies an arbitrary overhead rate of 100 percent to the $25.40 total hourly compensation to obtain a loaded hourly labor cost of $50.80. A sensitivity analysis will adopt the extreme factor of 0, implicitly ignoring indirect costs altogether.

For the simulation, a figure of 1.5 hours was used for the staff time involved with a certification. This figure represents a composite of figures from six Web sites at the county or state level that inform potential WIC applicants of how long they need to plan for conducting a certification. It is presumed that the 1.5-hour figure represents certification activity, as opposed to time spent in the provision of nonfood WIC benefits in the forms of health referrals, nutrition counseling, and breastfeeding promotion. Separate appointments at the WIC clinic are often made for these other activities.

Although the time for staff is assumed to be 1.5 hours for certification, the time for a client is assumed to be two hours to take into account travel time. About three-quarters of WIC households earn a wage or salary. Even those who have no labor income place a value on their time. An arbitrary figure of $6 an hour is used to estimate the value of time to the person—presumably the mother—who brings the WIC child to the program for certification. An arbitrary figure of $5 per certification is used for out-of-pocket travel costs for the WIC child’s mother.

Social Discount Rate

There is a literature on what the concept of the social discount rate means, or ought to mean, and how to measure it in terms of an observable private market discount rate. A range of 2 to 10 percent (in real terms) may encompass most estimates. The OMB annually provides federal agencies with updated discount rates, in support of Circular A-94 (OMB 1992), which provides guidance on benefit-cost analysis. As of January 2006, real discount rates for use in cost-effectiveness analysis (as opposed to other decisions) were reported as ranging from 2.5
percent for a three-year horizon up to 3.0 percent for a 30-year horizon (OMB 2006). The simulation uses a monthly discount rate of 0.002 for the baseline and an alternative figure of 0.004 for sensitivity analysis.

**Excess Burden**

OMB Circular A-94 gives a figure of 25 percent for \( \epsilon \), excess burden per dollar of taxes, although it also provides for the use of other figures. The 25 percent figure exceeds an estimate of 19.5 percent efficiency cost for combined local, state, and federal level taxes relative to a nondistortionary, revenue-neutral tax in a study by Jorgenson and Yun. Their study was reported to be one of the two studies with the broadest scope in a U.S. Government Accountability Office review of the compliance and efficiency cost of taxes (USGAO 2005). The simulation will use the 25 percent OMB figure to represent excess burden for the baseline, and an alternative figure of 19.5 percent.

**Simulations of Optimal Certification Durations for WIC Children**

Table 9.4 shows the results from the baseline simulation and sensitivity analysis. The figure of $112.25, the baseline model’s figure for recertification costs \( c \), is the sum of taxpayer and client costs. The estimated budgetary costs for recertification reflect $25.40 an hour for direct staff cost, a 100 percent overhead factor to obtain loaded labor costs, and an estimated time of 1.5 hours for a recertification, resulting in a total of $76.20 in budgetary terms. Taking into account an excess burden of taxation figure of 25 percent, the cost to the taxpayer of a recertification is $95.25. The cost to the client is estimated at $17, reflecting $12 worth of time costs ($6 an hour and two hours of time, including the time of the recertification and travel time) and $5 out-of-pocket travel costs. Social cost per recertification of \( [(1+\epsilon)c_d + c_c] \) totals $112.25 under the baseline.

The baseline figure for excess burden \( m \) is $8.13, equaling \( \epsilon M \) of (0.25) \times ($32.52).

Columns 3 and 4 show the solution to the baseline simulation as a pair of values, \( T^* \) and its associated \( M(T^*) \), representing the optimal, cost-minimizing certification duration and the minimized value of total cost that results from using \( T^* \). The baseline simulation results in \( T^* \) of
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Table 9.4 Results of CD Model Simulations

<table>
<thead>
<tr>
<th></th>
<th>Recert. cost ($)</th>
<th>Excess burden εM ($)</th>
<th>Optimal T* (end-of-month)</th>
<th>Optimal M(T*) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline model</td>
<td>112.25</td>
<td>8.13</td>
<td>12 months</td>
<td>486.28</td>
</tr>
<tr>
<td>Change from baseline:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>double M</td>
<td>112.25</td>
<td>16.26</td>
<td>9 months</td>
<td>540.90</td>
</tr>
<tr>
<td>overhead factor of 0</td>
<td>64.63</td>
<td>8.13</td>
<td>9 months</td>
<td>304.09</td>
</tr>
<tr>
<td>ε decreased to 19.5 percent</td>
<td>108.06</td>
<td>6.34</td>
<td>13 months</td>
<td>455.40</td>
</tr>
<tr>
<td>ρ raised to 0.4 per month</td>
<td>112.25</td>
<td>8.13</td>
<td>16 months</td>
<td>442.10</td>
</tr>
<tr>
<td>low income volatility</td>
<td>112.25</td>
<td>8.13</td>
<td>14 months</td>
<td>492.23</td>
</tr>
<tr>
<td>high income volatility</td>
<td>112.25</td>
<td>8.13</td>
<td>7 months</td>
<td>474.58</td>
</tr>
</tbody>
</table>

SOURCE: Author’s analysis.

12 months and M(T*) of $486.28. This solution occurs in Figure 9.1, which is depicted using the baseline values. Other parameter values used in the sensitivity analysis would change the locations and shapes of the curves, resulting in new, numerically different solutions T* and M(T*).

Sensitivity Analysis: Monthly Benefits

The first sensitivity analysis considers how an optimal certification duration might change if a different figure were used for monthly WIC benefits M and all other parameters were fixed at their baseline values. While the baseline figure for M was $32.52 (the 17-state average), the WIC children’s food package cost in New York was estimated to be $36.27 in Table 9.3. Using the New York figure—the highest among the 17 states in the table—results in a value for m of $9.0675 and for T* of 11 months (not shown in Table 9.4), which is shorter than the baseline’s 12. It makes sense that T* is now shorter because it pays to recertify more frequently when the potential savings from terminating unwarranted benefits is greater.
The $3.75 increase in \( M \) to the New York figure is about 10 percent of the baseline value. The increase did lower \( T^* \) by a month; however, such effects are not linear: increasing \( M \) by a steady amount does not lower \( T^* \) by one month in steady increments. The curve for recertification costs steepens sharply as certification duration is lowered, and that factor becomes increasingly difficult to overcome. Suppose instead that monthly benefits \( M \) were to double, to $65.04—a change in children’s WIC food package cost that far exceeds the $3.75 increase (and is far outside the 17-state range). Table 9.4 shows that an increase in effective benefits of this magnitude results in a decrease in \( T^* \) to nine months, down by just three months from the baseline case.

A doubling of the effective benefit is not simply a sensitivity analysis that gauges a relationship between \( T^* \) and \( M \). While no state is known to have a WIC food package cost as high as $65.04 per package—i.e., per child—in every state there are WIC households in which two WIC children (ages one to four) reside. While WIC certifies and provides benefits at the level of the individual, a household with two WIC children can receive double the food package benefits received by a household with one WIC child. An imaginable alternative to existing certification policy is that the WIC certification period for a WIC household could depend on the number of children (and, more generally, the number of other WIC participants) in the household.

**Sensitivity Analysis: Overhead Rate**

In the baseline, an arbitrary overhead rate of 100 percent is applied to total compensation (per hour) as part of estimating the WIC agency’s cost of recertification. If no overhead factor is applied, the budgetary cost per certification would drop by half, to $38.10 \( \left( \frac{25.40}{\text{hr}} \right) \left( 1 \frac{1}{2} \text{ hr} \right) \). As would be expected, \( T^* \) drops below the baseline figure of 12 months. The new \( T^* \) is nine months, equaling the \( T^* \) from the simulation that considered doubling \( M \).

Drawing on the result in the previous section that \( T^* \) depends on the relative values of \( c \) and \( m \) through the term \( \left( \frac{pc}{m} \right) \), it would be expected that dropping \( c \) by half would necessarily result in exactly the same estimated \( T^* \) as doubling \( m \) by doubling \( M \). However, \( c \) has not quite fallen by half: \( c \) is the sum of both \( c_A \) and \( c_C \), and \( c_C \) is unchanged (at $17).
the end, though, the change in $c$ is close enough to half to yield the same nine-month figure in both sensitivity analyses.

**Sensitivity Analysis: Excess Burden**

As the value for $c$ varies, everything else being equal, excess burden $cM$ will vary in proportion but recertification costs $[(1+c)c + c_c]$ will vary less than proportionately because the component of the recertification costs paid by the client is unaffected by variation in $c$. When reducing $c$ from the OMB figure of 0.25 to the Jorgenson and Yun figure of 0.195, the reduction in percentage terms for excess burden from $8.13 to $6.34 is relatively large, while the reduction in recertification cost from $112.25 to $108.06 is relatively small, making the $(pc/m)$ necessarily rise. The resulting increase in $(pc/m)$ drives $T^*$ to increase from the baseline’s 12-month $T^*$ to a new $T^*$ of 13 months. With lower values for both excess burden and recertification costs, the new $M(T^*)$ drops from the baseline (to $455.40$).

**Sensitivity Analysis: Discount Rate**

Doubling the social discount rate from the baseline figure of 0.2 percent per month to 0.4 percent per month increases $T^*$ from 12 months in the baseline to 16 months. $M(T^*)$ drops from $486.28 to $442.10 when future values are discounted more.

**Sensitivity Analysis: Income Volatility**

The pair of income volatility parameters $(\lambda, \mu)$ in the baseline simulation are $(0.07725, 0.23175)$, derived from an ordinary least squares (OLS) best fit of the Newman data. If instead the parameters are selected to provide upper and lower bounds to the three-year average figures of the state probability path $P_e(t)$, different estimates for $T^*$ would result. Retaining the stipulation that the state probability path has a steady state at 0.25 (which is $\lambda/\lambda + \mu$ in the model) means that $\mu$ will be three times the value of $\lambda$ for any selected $\lambda$. Thus, this sensitivity analysis is not being conducted as an exercise in which one parameter is varied and all others are held constant. Income volatility is examined here through the joint variation of $\lambda$ and $\mu$. 
Figure 9.3 shows two smooth state probability paths, a low path and a high path, that sandwich the three-year average state probability path of the Newman data. The low path is generated by $\lambda = 0.06$ (and $\mu$ of 0.18), while the high path is generated by $\lambda = 0.14$ (and $\mu$ of 0.42). The low path is associated with low income volatility (i.e., low parameter values for the two income volatility parameters) as well as low values of $P_E(t)$ at any given $t$ along the path. Correspondingly, the high path is associated with high income volatility and high values of $P_E(t)$.

Table 9.4 shows the results for a scenario that uses the baseline parameters except for income volatility, for which low values are used. $T^*$ increases from 12 months at baseline to 14 months with low income volatility. Under high income volatility, $T^*$ is seven months.

NOTE: Families with school-aged children become ineligible at 185% of the poverty line.

SOURCE: Table 9.2 and author’s analysis.
CONCLUSION

The optimal certification duration model shows in algebraic and graphical terms two policy trade-offs faced by program administration. On the one hand, there is economic savings to be had from detecting ineligible clients and terminating unwarranted benefits, thus saving the excess burden of taxation that is used to finance the benefits. Recertification is an administrative tool by which to determine whether a client is currently eligible. More frequent recertifications can reduce the excess burden of unwarranted benefits. At the same time, though, more frequent recertifications entail economic costs paid by WIC (ultimately, taxpayers) and by clients. In optimal decision-making, the costs of staff and equipment, of time and travel, are balanced against the costs of the excess burden of unwarranted benefits.

The optimal certification duration model takes into account the probabilistic nature of income dynamics and the time-dependency of probability paths of exit from eligibility and reentry into eligibility. It discounts future benefits and costs. It recognizes that there are repercussions to the likelihood and outcomes of future recertifications from conducting any given recertification.

Despite its strengths, the optimal certification duration model is a simplified rendition of the recertification problem that made use of workable functional forms (the exponential distribution) and an infinite-horizon specification. There is another limitation of the model that merits recognizing. The model focuses on the trade-off between a pair of program goals—specifically, benefits targeting and recertification costs. Another program goal that was set aside by the model is client access, which refers to sustaining participation by eligibles. Each recertification is a burden on the client, entailing monetary and time costs. Frequent recertifications may act as a barrier or a disincentive to program participation, potentially decreasing participation by some of the very clients whom the program was established to support. Thus, a longer certification duration may provide a social value in terms of improved client access. If so, then the certification durations estimated in the simulations may be interpreted as lower bounds of those that would be obtained from a fuller model that includes client access.
In one respect, introducing client access into the model is easy: simply define a “cost of client access” function, say $A(T)$, that would be a positive function of $T$, and add it to get a new total cost curve and a new cost-minimizing $T^*$. However, to conduct a simulation would now require figures for $A(T)$, figures that in turn require knowledge of clients’ behavior—knowing by how much participation falls as $T$ varies—and a figure for the social value of participation by eligibles.

The best-guess values of key parameters resulted in a baseline simulation of 12 months for the optimal certification duration $T^*$. From the theoretical model, it could be expected that an increase in WIC benefits or higher income volatility would lower $T^*$, while increases in agency or client recertification costs, excess burden per dollar of taxes or lower income volatility would increase $T^*$. A reason for conducting numerical simulations is to gauge the sensitivity in numerical terms of changes in $T^*$ to changes in parameters. $T^*$ varied from 7 to 14 months, depending on the simulation. Because some WIC households have more than one child receiving benefits, the model suggests that optimal $T^*$ could depend on the amount of WIC benefits any one household is receiving rather than on using a common certification duration for all WIC children.

An economic model can help clarify why different voices in the political process have different recommendations. The optimal certification duration model shows that there are many factors that affect certification durations. Different recommendations on certification durations would naturally follow from different notions of the size or strength of these factors that are in the model, as well as from different assessments of how important are the factors—such as client access—that have been left out of the model. Data and economic analysis can serve to quantify factors that may otherwise be impressionistic. The contribution of an economic model serves to help provide a common framework for analysis and discussion.

Every model has strengths and limitations; no model is complete. The theoretical and simulation results here are not presented as definitive, but as a first exploration of an issue of importance to policymakers, to program clients, and to the many other stakeholders with interests in the operations and effects of USDA food assistance programs.
Notes

I would like to acknowledge that this work benefited from comments by Dean Jolliffe, James Ziliak, Peter Gottschalk, Michael LeBlanc, David Smallwood, and Margaret Andrews. The views expressed here are those of the author and not necessarily those of the Economic Research Service or the U.S. Department of Agriculture.

1. To be eligible for WIC, a client must also be considered by a health professional to be at nutritional risk. Few applicants do not meet that criterion. Here I focus on income as if it were the sole determinant of WIC eligibility.


3. For fiscal 2005, the annual participation figures were 1,966,249 women, 2,047,118 infants, and 4,009,248 children (USDA 2008b).

4. As noted by Jorgenson, McCall, and Radner, “For the exponential distribution of time to failure . . . inspection, like replacement, serves as a point of regeneration of the investment process. This property of the exponential distribution, often referred to as the Markovian property or “lack of memory,” is not shared by any other distribution of time to failure” (p. 90).

5. The parameter \( \mu \) is a transition rate of reentry for clients who had previously exited from eligibility, as opposed to a hazard rate of initial entry for the general population of ineligibles. Formerly eligible clients constitute a group that can be expected to exhibit (a) lower income than the general ineligible population and (b) a higher rate at which eligibility is (re)entered.

6. The certification duration model makes repeated use of (9.10) and (9.11) and for brevity calls them “state probabilities” even though they are properly understood as “conditional” in contrast to (9.8) and (9.9).

7. There is another pair of probability paths (not shown) for the probabilities that the client is either eligible or ineligible at time \( t \), given that the client is ineligible at time zero; these two paths are not used here.

8. It is this feature of the model that would be absent if hazard rates were non-constant.

9. Intuition suggests that the exponent on the \( P_{EE}(T) \) term in Equation (9.25) should be \( n - 1 \) rather than \( n - 2 \). However, counting the number of passed recertifications to reach the nth cycle differs from the usual count of “successful trials” in statistical theory. WIC is examining the optimal certification duration for a client who has just been determined to be eligible at the initial application, thus removing one \( P_{EE}(T) \) (for the initial application) from consideration and changing the exponent by 1.


11. It is known from earlier studies that income is volatile for women surrounding the
birth event, raising the question of how well income dynamics match for households with WIC children and households with school-aged children. However, research findings suggest that many mothers who had been employed before pregnancy return to work within the first year after giving birth. Klerman and Leibowitz (1990) find that over one-third of mothers were at work within three months following birth, and three-quarters within two years. The simulations here presume that the volatility surrounding the birth event is sufficiently resolved by the end of the first year that the birth event does not substantially affect income dynamics of households with WIC children (ages one to four).

12. One small difference in labels is noted: the first month in the table is month 0, while in the figure it is month 1.

13. A recent study on WIC staffing states that “little specific information exists about the actual performance of duties throughout the various classifications within the nutrition workforce, or specifically the WIC workforce” (USDA 2006b, p. 31). The same study notes that “Some clinics attempt to get both anthropometric and blood work from other providers, while some clinics do their own anthropometrics and use blood work obtained elsewhere” (p. 21).

14. A weighted-average approach would draw upon such information as a 1999–2000 survey of staffing and annual salaries reported by the USDA (2003), in which the range between the median low salary and the median high salary for various positions was $18,804–$25,251 for nutrition assistant, $20,736–$29,163 for nutrition technician, $26,352–$39,000 for nutritionist, and $29,661–$43,496 for clinical nutritionist; five other job titles and salaries were reported, too. One difficulty with constructing weighted-average wage rate is that positions with these titles have nonuniform duties across local WIC agencies. Another is the absence of information on the time each contributes to a certification.

15. A check on the correspondence between the hourly wage of an actual WIC staff position and the BLS figure is provided by a job-vacancy notice posted in summer 2006 by Rice County, Minnesota, seeking a WIC professional—a public health nurse or nutrition professional (Rice County 2006). The job activities included “all phases of the WIC certification process, including hematological screening.” The stated hourly salary range was $19.31–$26.71, for which the midpoint is $23.01, which differs from the BLS figure by about one half-dollar.

16. Quarterly data for 2005 are available for both “health care and social assistance employees” and for “all occupations” (both series in the state and local government sector). Total compensation was computed for each of four quarters of 2005 for the two groups using “cost per hour worked” and “percent of total compensation.” The annual average of the quarterly figures is $35.92 for “all occupations” and $33.42 for “health care and social assistance.” In 2005, “health care and social assistance” workers received (on average) 93.0 percent of the total compensation paid to “all occupations.” Applying the 93.0 percent figure to the total compensation for “all workers” in 1998 of $27.30 yields a figure of $25.40 for “health care and social assistance employees” in that year.

17. The six Web sites were selected based only on convenience, rather than for constituting a nationally representative sample. The reported figures were “about”
30 minutes in Minnesota (MDH 2008); 1–2 hours in Michigan (MDCH 2008); 1–2 hours in Fairfax County, Virginia (FCHD 2008); “at least” 1½ hours in Clinton County, New York (CCHD 2008); “at least” 1½ hours in Utah County, Utah (UCHD 2006); and 1½–2 hours in St. Charles County, Missouri (SCCDPH 2008).

18. Of those households with WIC children, median annual income (all sources) was $15,325 for households with an adult male present and $8,520 for households without an adult male present in 1998 (USDA 2001, p. 76, Exhibit 3-22). The simulations take the $8,520 figure as the annual income (all sources) of a mother with a WIC child with or without an adult male present. The assumption of $6 per hour is consistent with annual earnings of $6,000 and 1,000 hours of annual work.

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2008

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