A Model of Fringe Benefit Provision

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The mix of fringe benefits and wages in the compensation package depends jointly on the decisions of employers and employees, both of whom operate under two sets of constraints. The first set of constraints can be thought of as market constraints associated with factor prices, goods prices, technology, and incomes. The second can be thought of as constraints (or incentives) established by government, with tax policies and the Employee Retirement Income Security Act (ERISA) being the two outstanding examples. Thus, although the employer actually provides the compensation package, that package must be fashioned subject to the preferences of workers (who can alter their level of effort or seek employment elsewhere if a compensation package is unsuitable) and subject to government policies (for example, meeting ERISA vesting, funding, fiduciary, and reporting standards if a defined-benefit pension plan is part of the compensation package). All these factors then—workers' preferences, employers' costs, and government policy—must be a part of any overarching theory of fringe benefit determination.

It is convenient to model the determination of fringe benefits by supposing that the employer offers a menu of possible benefit packages—or at least is willing to offer a variety of benefit packages—and that workers select the package they most prefer. Institutionally, this may seem somewhat unrealistic, because we normally picture the employer making the benefit determination unilaterally—the worker must take or leave whatever is offered. But employers do fashion their benefit packages with the preferences of workers in mind; indeed, if they failed to do so, they would find themselves with a level of labor turnover higher than desired, or a workforce of lesser quality than desired. An important advantage of casting the problem in this way is that it allows us to make use of the large theoretical and applied literature on consumer theory,
which is one of the main developments of modern economics (see, for example, the excellent texts by Deaton and Muellbauer 1980b; Phlips 1983; and Theil 1980). So, although modeling fringe benefits as “chosen” by workers may seem somewhat inconsistent with the institutional realities of the process of benefit provision, it does little violence to our ability to predict benefit outcomes as long as we take full account of the constraints and costs faced by the employer in deciding what benefit packages are acceptable.

All existing research that attempts to model explicitly the mix of compensation approaches the problem as one where employees maximize utility by selecting from among a variety of possibilities presented by the employer (see Alpert 1983; Freeman 1981; Sloan and Adamache 1986; Smeeding 1983; Triplett 1983; and Woodbury 1983, for example). The reason is that much of this existing work has focused on the responses of workers to the favorable tax treatment that nonwage benefits receive under the federal personal income tax. There is no convenient way of modeling this aspect of benefit determination through an approach that focuses on the employer. However, nothing in the utility-maximization approach commonly used precludes complete consideration of the factors determining the employer’s willingness to offer benefits.

In the remainder of this chapter, a model developed previously to describe the tradeoff between wages and fringes (Woodbury 1983) is extended to include tradeoffs between a pair of fringe benefits. The model developed is a utility-maximizing model that restricts attention to the determination of wages and the two most important voluntarily provided fringe benefits: pension plans and insurance contributions (most of which are for health care). Employer contributions to social security, unemployment insurance, and workers’ compensation are not determined within the model because provision of these benefits is legally mandated and beyond the scope of the firm’s decision. Neither is the consumption by workers of goods such as food and clothing determined within the model because no available data set includes data on both goods consumption and fringe benefit consumption.
Heuristic Statement of the Model

Consider that there are only three forms of compensation: wage benefits, pension benefits, and health insurance benefits. Consider also that the worker's utility depends on the quantities of goods bought with wage benefits ($z_w$), pensions or retirement benefits ($z_r$), and health insurance benefits ($z_h$):

$$U = U(z_w, z_r, z_h).$$

A worker chooses quantities of $z_w$, $z_r$, and $z_h$ so as to maximize utility subject to a budget constraint that can be written:

$$z_w p_w + z_r p_r + z_h p_h = m.$$ 

In equation (2.2), $p_w$, $p_r$, and $p_h$ are the prices of wage goods, pension or retirement benefits, and health insurance benefits, each of which is beyond the worker's control. Also, $m$ is the maximum dollar amount that the employer is willing to expend on workers' wages, pensions, and health insurance. Although $m$ excludes various components of compensation such as social security and in-kind perquisites, it will be referred to as total compensation in what follows.

Stated in this way, the problem of fringe benefit choice is a straightforward problem of consumer demand and may be handled by well-known techniques. But two basic difficulties need to be overcome. First, we do not ordinarily observe quantities of the components of compensation ($z_w$, $z_r$, and $z_h$) or the prices of those components ($p_w$, $p_r$, and $p_h$). We do observe the dollar amounts expended by employers on the various components of compensation. For example, $p_w z_w$ would be the number of dollars paid to a worker in wages, and $p_r z_r$ would be the dollar contribution to the worker's pension plan. But prices and quantities must be observed separately in order to estimate some representation of the utility function (eq. 2.1). Accordingly, if the consumer model is to be made workable in this application, some effort will need to be spent identifying and measuring either the quantities or the prices of the...
components of compensation. Measurement of these prices is discussed further in a later section of this chapter and in the appendix to chapter 3.

The second difficulty to overcome is that the budget constraint (eq. 2.2) turns out to have some unconventional features that require careful treatment. In discussing these unconventional features, it is useful to proceed in two steps. First, we examine the tradeoff between wages and either of the two fringe benefit goods—pensions or health insurance. This allows us to focus on the fact that wages are taxed, whereas fringe benefits are not, and to draw out the implications of this difference in tax treatment. Second, we examine the tradeoff between pensions and health insurance. This allows us to focus on the differences in the employer's cost of providing different fringe benefits, and to draw out the implications of these cost differences for fringe benefit demand.

**Wage Taxation and the Tradeoff Between Wages and Fringe Benefits**

Suppose for the moment that the quantity of health insurance benefits has already been determined and is held fixed. This allows us to analyze substitution between wage benefits and pension benefits in a two-dimensional diagram such as figure 2.1, which depicts a representative worker's preferences for pension and wage benefits. The quantity of wage benefits ($z_w$) is measured along the horizontal axis, and the quantity of pension benefits ($z_r$) along the vertical axis. $U_0$ and $U_1$ are indifference curves, each showing combinations of wage and pension benefits that yield a given level of utility.

If the employer is willing to spend a total of $m_1$ dollars in compensating the worker with wage and pension benefits and is willing to exchange wages for pensions at a rate of \( \frac{c_w}{c_r} \), then the employer will offer the worker any combination of wages and pensions that lies along the locus:

\[
z_w c_w + z_r c_r = m_1.
\]  

(2.3)

Note that $c_w$ is the employer's cost of providing wages, and $c_r$ is the employer's cost of providing pensions. Note also that the dollar amount $m_1$ equals total compensation ($m$) minus the predetermined expenditure on health insurance.
Figure 2.1. The optimal mix of wages and pension benefits shifts from \((z_w^*, z_r^*)\) without income taxation to \((z_w^{**}, z_r^{**})\) with an income tax. 

\(z_w\) denotes the quantity of wages, \(z_r\) denotes the quantity of pension benefits.

Given this initial budget constraint, the worker would maximize utility by choosing a quantity of wages, \(z_w^*\), and a quantity of pensions, \(z_r^*\), as shown in figure 2.1.

The initial constraint described by equation (2.3), however, is not the constraint that the worker finally faces. Under the current tax system, the worker's wages would be taxed at some marginal rate \((t)\) whereas the employer's contributions to the worker's pension benefits would go untaxed. It follows that the budget constraint facing the worker pivots to

\[z_w p_w + z_r p_r = m_1\]  

(2.4)

and the worker maximizes utility subject to this tax-modified constraint. The new optimal bundle of wages and pensions is given by \((z_w^{**}, z_r^{**})\).
Note that \((p_w/p_r)\) is the rate at which the worker is able to exchange wages for pensions; that is, \(p_w\) and \(p_r\) are the tax-modified prices of wages and pensions. The relation between \((p_w/p_r)\), which is the rate at which the worker is able to exchange wages for pensions, and \((c_w/c_r)\), which is the rate at which the employer is willing to trade wages for pensions, is a function of the marginal tax rate:

\[
p_w/p_r = (c_w/c_r)/(1-t). \tag{2.5}
\]

If \(c_w = c_r\), equation (2.5) can be simplified to:

\[
p_w/p_r = 1/(1-t). \tag{2.6}
\]

An important simplification that has been incorporated into figure 2.1 is that the marginal tax rate \((t)\) is taken as constant. In fact, of course, the marginal tax rate facing a worker varies with the income of the worker and his or her spouse. This variation in the marginal tax rate is both a blessing and a bane. It is a blessing because variation in \(t\) means that we will observe variation in the price ratio \((p_w/p_r)\) facing different individuals or groups in any sample where there is variation in taxable income. Such variation will be central to estimating some representation of the utility function (eq. 2.1). Further, because marginal tax rates are a discontinuous function (or step function) of income, it should be possible to separate the effects of changes in income from the effects of changes in the relative price of wages and pensions. That is, income and the price ratio do vary independently; hence, econometric estimation of both substitution effects (which could also be thought of as tax effects) and income effects should be possible.

The variation of the marginal tax rate with income is a bane because this variation makes the budget constraint facing the worker nonlinear, rather than linear as shown in figure 2.1. That is, as the share of total compensation received as wages increases, the marginal tax rate will increase. In figure 2.1, the budget constraint labeled \(z wp_w + z rp_r = m_1\) should bend toward the wage axis and intercept the wage axis at some point to the left of \(m_1/p_w\). The nonlinearity of the budget constraint has econometric implications that are discussed below.
Fringe Benefit Costs and the Tradeoff Between Pensions and Health Insurance

Suppose now that the quantity of wage benefits has been predetermined and is held fixed, so that we may show the tradeoff between pensions and health insurance benefits in a two-dimensional graph. Figure 2.2 is such a graph; it shows the representative worker's preferences, this time for pension benefits ($z_r$, shown on the horizontal axis) and health insurance benefits ($z_h$, shown on the vertical axis). The indifference curve $U_0$ shows combinations of pension and health insurance benefits that give the worker equal satisfaction.
The key to analyzing substitution between pensions and health insurance benefits is to note that the employer's cost of providing pensions differs from the cost of providing health benefits. If the employer is willing to spend a total of $m_2$ dollars in compensating this worker with pensions and health insurance, and if the firm must pay $c_r$ per unit of pension benefit, and $c_h$ per unit of health insurance, then the firm will offer this worker any combination of pensions and health benefits lying on the locus:

$$z_r c_r + z_h c_h = m_2.$$  \hspace{1cm} (2.7)

The dollar amount $m_2$ equals total compensation ($m$) minus the predetermined expenditure on wages. The dollar expenditures on pensions and health benefits are the two terms on the left-hand side of equation (2.7).

In the absence of any taxes on either fringe benefit, the worker will choose the fringe benefit package ($z_r^*, z_h^*$). A different employer, however, might face different costs of providing pensions and health benefits. For example, a small employer might face higher per unit insurance costs than a large employer (so that the cost of health benefits now equals $c'_h$), and might be willing to make a total dollar expenditure on pensions and health insurance equal to $m_3$, which would yield the set of pension-health offerings depicted by $z_r c_r + z_h c'_h = m_3$. In this latter case, the worker chooses a different fringe benefit package ($z_r^{**}, z_h^{**}$), with more pension benefits and less health insurance.\footnote{Note also that if a given employer faced rising health insurance costs over time, the initial constraint would pivot, and a similar substitution of pensions for health benefits would be likely to result.}

**A More Formal Statement of the Model**

Recall our basic supposition that the worker's utility depends on the *quantities* of wage benefits ($z_w$), pensions or retirement benefits ($z_r$), and health insurance benefits ($z_h$):
Empirical estimates of this utility function would yield information about how substitutable are these three components of compensation for one another. Such information could, in turn, be used to estimate how a change in tax policy that favored one component of compensation would affect demand for all three components of compensation. In effect, we are trying to estimate the shapes of the indifference curves shown in figures 2.1 and 2.2, and to use those estimates to predict how much substitution of one form of compensation for another would occur in response to a shift of the budget constraint.

Elasticities to Be Estimated

More precisely, we seek estimates of the following elasticities:

1. The Uncompensated Own- and Cross-Price Elasticities of Demand for Benefit i:

   \[ \eta_{ij} = \left( \frac{\partial z_i}{\partial p_j} \right) \frac{p_j}{z_i}. \]  

   This is the percentage change in demand for benefit i that can be expected in response to a 1 percent change in the price of benefit j. \( \eta_{ij} \) is often referred to as the Marshallian price elasticity of demand.

2. The Compensated Own- and Cross-Price Elasticities of Demand for Benefit i:

   \[ \eta_{ij}^* = \left( \frac{\partial z_i}{\partial p_j} \right) \frac{p_j}{z_i}. \]  

   This is the percentage change in demand for benefit i that can be expected in response to a 1 percent change in the price of benefit j, holding utility constant. That is, \( \eta_{ij}^* \) measures changes in demand that take place after the worker has been compensated for any changes in utility that occur as a result of the price change. \( \eta_{ij}^* \) is often referred to as the Hicksian compensated elasticity of demand.
3. The Income Elasticity of Demand for Benefit $i$:

$$\eta_{im} = (\frac{\partial z_i}{\partial m})(m/z_i).$$  \hspace{1cm} (2.10)

This is the percentage change in demand for benefit $i$ that occurs in response to a 1 percent change in total compensation.

4. The Elasticity of Substitution between Benefit $i$ and Benefit $j$:

$$\sigma_{ij} = \frac{\eta_{ij}^*}{s_j}$$  \hspace{1cm} (2.11)

where $s_j$ is the share of total compensation received as benefit $j$. This is a measure of the strength and type of relationship (substitutability or complementarity) between benefits $i$ and $j$. It can be either positive or negative. If $\sigma_{ij}$ is positive, the benefits are substitutes; if negative, they are complements.

These four elasticities are tied together by the Slutsky relation, which in elasticity form can be written:

$$\eta_{ij} = \eta_{ij}^* - s_j \eta_{im}$$  \hspace{1cm} (2.12)

or, by substituting equation (2.11) into equation (2.12),

$$\eta_{ij} = \sigma_{ij} s_j - s_j \eta_{im}.$$  \hspace{1cm} (2.13)

Estimates of these elasticities will provide the information that is needed to predict the effects of various tax changes on the demand for each form of compensation.

A System of Demand Equations for the Components of Compensation

Our goal, then, is to estimate some representation of the utility function (eq. 2.1), so as to obtain an unrestricted set of price, income, and substitution elasticities as defined by equations (2.8) through (2.11) above. The problem is that utility ($U$) is unobservable, so that a utility function such as equation (2.1) cannot be estimated directly. The most straightforward way of solving this problem is to manipulate or transform the direct utility function into a form that permits estimation and
yields the same information about demand and substitution possibilities. Duality theory allows one to make just such manipulations. It turns out that if the direct utility function (eq. 2.1) is well-behaved, then an indirect utility function, dual to it, can be written showing the maximum utility attainable by a worker facing a price of wage benefits \( (p_w) \), a price of retirement benefits \( (p_r) \), a price of health benefits \( (p_h) \), and a given level of total compensation \( (m) \):

\[
V = V(p_w, p_r, p_h, m).
\]  \hspace{1cm} (2.14)

Further, the indirect utility function may be solved for the minimum expenditure or cost \( (C) \) required to attain a specified level of utility \( (U) \), given prices \( (p_w, p_r, \) and \( p_h) \). So rewritten, equation (2.14) becomes a consumer cost function (or expenditure function):

\[
C = C(p_w, p_r, p_h, U).
\]  \hspace{1cm} (2.15)

Since our goal is to estimate an unrestricted set of elasticities of substitution for wages, pension benefits, and health benefits, it is desirable to estimate an arbitrary approximation to either the indirect utility function (eq. 2.14) or the consumer cost function (eq. 2.15) set out above. For example, a translog approximation to the indirect utility function could be estimated, as it has been in earlier work on similar issues (Woodbury 1983, 1985a). Although satisfactory, the translog indirect utility function requires costly nonlinear estimation techniques unless one is willing to restrict income elasticities to unity and estimate a homothetic indirect utility function. Since we do not wish to impose unitary income elasticities, it would be advantageous to find some alternative.

One attractive alternative is to estimate a representation of the consumer cost function represented by equation (2.15). Deaton and Muellbauer (1980a) have developed a flexible approximation to the consumer cost function that results in an easily estimated system of consumer demand equations. They start with a representation of the consumer cost function known as the PIGLOG class (for Price Independent Generalized Linear Logarithmic). For the case of three goods \( (w, p_w, p_r, p_h)$,
Deaton and Muellbauer’s consumer cost function can be written as:

\[
\ln C(p_w, p_r, p_h, U) = a_0 + a'_w \ln p_w + a'_r \ln p_r + a'_h \ln p_h + (\frac{1}{2})b'_{ww}(\ln p_w)^2 + (\frac{1}{2})b'_{wr}(\ln p_w)(\ln p_r) + (\frac{1}{2})b'_{wh}(\ln p_w)(\ln p_h) + (\frac{1}{2})b'_{rr}(\ln p_r)^2 + (\frac{1}{2})b'_{rh}(\ln p_r)(\ln p_h) + (\frac{1}{2})b'_{hh}(\ln p_h)^2 + U b_0 (p_w)^b_w (p_r)^b_r (p_h)^b_h.
\]

All variables in equation (2.16) have been defined previously, except for the parameters \(a_0, a'_i, b_{ij}, b_0, \) and \(b_i\). These parameters characterize preferences for the various forms of compensation and will be estimated econometrically. The parameter estimates in turn allow one to compute directly the price, income, and substitution elasticities set out in equations (2.8) through (2.11). (See below.)

Demand functions for wage benefits, pensions, and health insurance can be derived from equation (2.16) by differentiating with respect to \(p_w, p_r,\) and \(p_h\). This is the well-known property of cost functions developed by Shephard (1970). By some further manipulation, each demand function can be transformed into a demand function in budget-share form. For the consumer cost function represented by equation (2.16), these budget shares are:

\[
s_w = a'_w + b_{ww} \ln p_w + b_{wr} \ln p_r + b_{wh} \ln p_h + b_w \ln (m/P)
\]

\[
s_r = a'_r + b_{rw} \ln p_w + b_{rr} \ln p_r + b_{rh} \ln p_h + b_r \ln (m/P)
\]

\[
s_h = a'_h + b_{hw} \ln p_w + b_{hr} \ln p_r + b_{hh} \ln p_h + b_h \ln (m/P)
\]

where \(b_{ij} = (\frac{1}{2})(b'_{ij} + b'_{ji})\).

Equations (2.17), (2.18), and (2.19) are three expenditure share equations. They say that the share of total compensation received in each
form is a function of prices and an income term \((m/P)\). Specifically, \(s_w\) is the share of compensation received as wages, \(s_r\) is the share received as pension or retirement benefits, and \(s_h\) is the share received as health benefits. The price of wages \((p_w)\), the price of pension or retirement benefits \((p_r)\), and the price of health benefits \((p_h)\) are as defined earlier. The income term \((m/P)\) equals after-tax total compensation \((m)\) divided by a price index, \(P\).

There is an exact definition of \(P\) (Deaton and Muellbauer 1980a, p. 314), which leads to a specification requiring nonlinear estimation techniques. Because nonlinear techniques are expensive and unattractive, several researchers, including Deaton and Muellbauer, have approximated \(P\) as:

\[
\ln P^* = s_w \ln p_w + s_r \ln p_r + s_h \ln p_h
\]

where \(P^*\) is the approximation to \(P\). Using this approximation is attractive because it turns \(P\) into a predetermined variable, and hence gives us a model that is linear in parameters and relatively simple to estimate. Also, those who have used it (Deaton and Muellbauer 1980a; Anderson and Blundell 1983, 1984; and Kang 1983) have found \(P^*\) to be a good approximation to the exact definition of \(P\). Accordingly, the approximation will be used throughout this work.

### Implementing the Model

Deaton and Muellbauer call the demand system represented by equations (2.17) through (2.19) the Almost Ideal Demand System (or, rather inauspiciously, the "AIDS") because it is log-linear in prices and income once an approximation to \(P\) has been selected. Linearity makes the Deaton-Muellbauer system simpler to estimate than other flexible representations of consumer preferences, such as the translog or generalized Leontief. Indeed, linearity is the most significant advantage of the Deaton-Muellbauer demand system over other demand systems that are consistent with consumer theory and allow estimation of an unrestricted set of price, income, and substitution elasticities.
Adding Demographic and Other Variables to the Model

An additional advantage of the Deaton-Muellbauer system is that it easily accommodates variables other than prices and income that may influence the observed share of fringe benefits. To this point in the discussion, there has been no attempt to include such variables. But there can be little doubt that unbiased estimation of the demand elasticities of interest requires that we control for variables such as individual demographic characteristics.

How to include demographic and other variables in empirical demand analysis is a problem that has been the topic of considerable research (see, for example, Lau, Lin, and Yotopoulos 1978; Pollak and Wales 1981; and Lewbel 1985). The solution adopted here is the obvious one in the context of the Deaton-Muellbauer demand system. Since each budget share includes a constant term, it is natural to suppose that the constant shifts with demographic changes. If we make such an assumption, which is known as demographic budget share translation, then equations (2.17) through (2.19) can be rewritten as follows:

\[ s_w = a_w + b_{ww} \ln p_w + b_{wr} \ln p_r + b_{wh} \ln p_h + b_w \ln (m/P*) + \]
\[ d_w x_1 + \ldots + d_w x_K \]  
where \( x_1, \ldots, x_K \) represent the demographic (and other) variables that shift the budget shares, and the \( d_{ik} \) are coefficients representing the effects of changes in those variables on the shares. Also, the approximation \( P* \) has been substituted for \( P \). Note that use of \( P* \) results in a redefinition of the intercept terms \( (a_i) \) in equations (2.21) through (2.23), which equal \( a'_i - b_i (\ln g) \), where \( g \) is a scalar indicating how well \( P* \) approximates \( P \) (if \( g = 1 \), the approximation is exact). (For details, see Deaton and Muellbauer 1980a, p. 316.)
Restrictions of Consumer Theory and Stochastic Assumptions

It is desirable for the demand system represented by equations (2.21) through (2.23) to exhibit three properties that are implied by consumer theory. These are usually referred to as the adding-up, homogeneity, and symmetry properties, and are treated in turn.

Adding-up simply means that the shares of the budget spent on the components of compensation sum to one. In order to satisfy the adding-up property, the system represented by equations (2.21) through (2.23) must satisfy the following across-equation constraints:

\[ a_w + a_r + a_h = 1 \]  \hspace{1cm} (2.24)
\[ b_{ww} + b_{rw} + b_{hw} = 0 \]  \hspace{1cm} (2.25)
\[ b_{wr} + b_{rr} + b_{hr} = 0 \]  \hspace{1cm} (2.26)
\[ b_{wh} + b_{rh} + b_{hh} = 0 \]  \hspace{1cm} (2.27)
\[ b_w + b_r + b_h = 0 \]  \hspace{1cm} (2.28)
\[ d_{wk} + d_{rk} + d_{hk} = 0 \text{ for all } k. \]  \hspace{1cm} (2.29)

In practice, the adding-up constraint is imposed by constructing the data so that the budget shares sum to one for each observation, and so that the mean of each price and income variable is unity (hence the natural logarithm of each mean is zero). Since, as discussed presently, only two of the three share equations must be estimated, constructing the budget shares in this way will cause the adding-up restrictions to be satisfied automatically.

Homogeneity implies that a doubling of both prices and income would leave the demand for each component of compensation unchanged. For the system to be homogeneous (specifically, homogeneous of degree zero in prices and income), the following within-equation constraints must be imposed:

\[ b_{ww} + b_{wr} + b_{wh} = 0 \]  \hspace{1cm} (2.30)
\[ b_{rw} + b_{rr} + b_{rh} = 0 \]  \hspace{1cm} (2.31)
\[
b_{hw} + b_{hr} + b_{hh} = 0. \tag{2.32}
\]

Finally, symmetry means that the cross-substitution effects must be symmetric—that is, \( \sigma_{ij} = \sigma_{ji} \). Such symmetry can be imposed by constraining \( b_{ij} \) to equal \( b_{ji} \) across equations in the econometric estimation.

If we impose the adding-up, homogeneity, and symmetry constraints on equations (2.21) through (2.23), and append a random disturbance term to each, we obtain the following three share equations:

\[
s_w = a_w + b_{wr} \ln(p_r/p_w) + b_{wh} \ln(p_h/p_w) + b_w \ln(m/P^*) + d_{w1}x_1 + \ldots + d_{wk}x_K + u_w \tag{2.33}
\]

\[
s_r = a_r + b_{rr} \ln(p_r/p_r) + b_{rh} \ln(p_h/p_r) + b_r \ln(m/P^*) + d_{r1}x_1 + \ldots + d_{rK}x_K + u_r \tag{2.34}
\]

\[
s_h = a_h + b_{rh} \ln(p_r/p_h) + b_{hh} \ln(p_h/p_h) + b_h \ln(m/P^*) + d_{h1}x_1 + \ldots + d_{hK}x_K + u_h \tag{2.35}
\]

where the random disturbance terms \(-u_w, u_r, u_h\) are assumed to be normally distributed with zero mean.

Two points are worth noting. First, the price ratios used in equations (2.33) through (2.35) are the inverses of those stated as equations (2.5) and (2.6). This poses no problem—equations (2.5) and (2.6) simply define the inverses of the appropriate price ratios. Second, because the shares \((s_w, s_r, s_h)\) sum to one, only two of the above three equations are independent. As a result, estimation of equations (2.33) through (2.35) can be accomplished by arbitrarily deleting one of the three equations and performing an iterative version of Zellner’s seemingly unrelated regression procedure on the remaining two. (See Christensen and Manser 1976, 1977 for further discussion and references.) Results will be the same regardless of which equation is deleted, and will be maximum likelihood estimates.

**Prices of the Components of Compensation**

The demand system given by equations (2.33) through (2.35) is driven by variation in the relative prices of the components of compensa-
tion \([(p_r/p_w)\) and \((p_h/p_w)\)], and by variation in income \((m/P^*)\). As a result, it is important to understand the origin of these price ratios and the income term. A complete treatment of the construction of these variables can be given only in the context of the data being considered, but the following general points are in order.

Recall that the price of wage benefits relative to the price of pensions is:

\[
p_{w}/p_{r} = (c_{w}/c_{r})/(1-t)
\]  
(2.5)

where \(c_w\) is the employer's cost of wage benefits, \(c_r\) is the employer's cost of pension benefits, and \(t\) is the marginal tax rate on wages faced by the worker. That is, income taxation shifts the rate at which the worker can trade wages for pensions, since the worker's wages are taxed at some marginal rate \((t)\) but the employer's contributions to pensions are untaxed. Hence, wages have become more expensive relative to pensions as a result of income taxation.

Because the employer's contributions to health insurance are also untaxed, income taxation will shift the rate at which the worker can trade wages for health insurance benefits:

\[
p_{w}/p_{h} = (c_{w}/c_{h})/(1-t)
\]  
(2.36)

where \(c_h\) is the employer's cost of providing health insurance benefits.

Finally, income taxation will not affect the tradeoff between pensions and health benefits faced by the worker if all fringe benefits are untaxed. That is,

\[
p_{r}/p_{h} = c_{r}/c_{h}
\]  
(2.37)

or the rate at which the worker can ultimately trade health for retirement benefits is the same as the rate at which the employer can trade them.

Note that only two of the three price ratios represented by equations (2.5), (2.36), and (2.37) are independent. This is another way of seeing that only two of the three equations of the demand system must be estimated. Indeed, the only relative prices needed to estimate the demand system represented by equations (2.33) through (2.35) are \((p_{w}/p_{r})\) and \((p_{w}/p_{h})\).
How are the needed price ratios \((p_w/p_r)\) and \((p_w/p_h)\) to be obtained? If we are willing to assume that the cost of pensions relative to the cost of wages—that is, \(c_w/c_r\)—equals an arbitrary constant, then the price ratio \((p_w/p_r)\) can be written simply as:

\[ p_w/p_r = k/(1-t) \]  

(2.6a)

where \(k\) is the constant of proportionality between wage and pension costs. Since the only source of variation in \((p_w/p_r)\) is the marginal tax rate \(t\) it is harmless to set \(c\) equal to 1 and to compute:

\[ p_w/p_r = 1/(1-t). \]  

(2.6)

Hence, \((p_w/p_r)\) is computed as a simple transformation of the marginal tax rate. (We explain how measures of the marginal tax rate are obtained in the next chapter.)

It is possible that the ratio of wage to pension costs \((k)\) is not constant, but varies from industry to industry. The source of this variation can be thought of as exogenous interindustry differences in the organization of production, which lead to differing specific human capital requirements, turnover of workers, and pension provision. In our empirical specifications in the next chapter, we include proxies for the organization of production independently, rather than attempting to embed the effects of differing organization of production in the price ratio \((p_w/p_r)\).

The first reason for this independent inclusion is its feasibility and the apparent infeasibility of embedding the effect of differences in the organization of production in the price ratio \((p_w/p_r)\).

The second reason is that independent inclusion of a specific human capital measure offers a way of testing for so-called agency incentives for pension provision. As Lazear (1981), Mumy and Manson (1985), and Bell and Hart (1990) have pointed out, firms may offer deferred compensation as an inducement to their employees to work hard and remain with the firm over many years, since only employees who stay with the firm will receive their deferred compensation. It follows that deferred compensation such as pensions will be important in industries where the organization of production places a premium on skill and specific human capital. Accordingly, we include proxies for specific
human capital in our estimates of fringe benefit demand reported in the
next chapter, with the expectation that higher levels of specific human
capital will be related to a higher proportion of total compensation
received as pensions, but will be unrelated to the share of compensation
received as health insurance. 7

Estimates of \( p_w/p_h \) can be obtained by observing health insurance
costs, which can be observed both over time and from employer to
employer. Changes over time may be obtained from the price index of
health insurance reported in the National Income and Product Accounts
(U.S. Department of Commerce, Bureau of Economic Analysis 1986,
Table 7.10). Differences from employer to employer are more difficult
to measure. Health insurance carriers vary their rates depending on the
health and medical care experience of a group, which in turn depends on
the group’s size, occupational mix, and demographic composition.
Estimation of these variations is taken up in the next chapter.

To summarize, since variation in the price of wage benefits relative to
pension and health benefits can be observed, it should be possible to
estimate tradeoffs between wages and pensions, between wages and
health insurance, and between pensions and health insurance. The
tradeoff between pensions and health insurance is of special interest.
First, it has not been estimated in previous research. Moreover, pro-
posals have been advanced to tax health insurance contributions but not
pension contributions under the federal personal income tax. Determin-
ing the effect of such a tax policy on pension provision requires under-
standing of the possibilities for substituting pensions for health insur-
ance. The assumption made in all previous analyses of the provision of
fringe benefits has been that employers’ costs of providing a unit of each
benefit \( c_w, c_r, \) and \( c_h \) are equal. Only by relaxing this assumption can
we consider tradeoffs within the fringe benefit package, such as the
tradeoff between pensions and health insurance.

**Progressive Taxes and Nonlinearity of the Budget Constraint**

Recall that because the marginal tax rate facing a worker increases as
his or her income increases, the tradeoff between wages and fringe
benefits facing that worker will be nonlinear. That is, the lower of the two budget constraints in figure 2.1 should actually bend toward the wage axis as the worker consumes more wages and fewer fringe benefits.

In fact, the nonlinearity of the budget constraint in the context of fringe benefit demand is a special case of a more general problem that results from progressive taxation. For example, the budget constraint facing workers when they make their labor supply decision is also nonlinear, because as a worker's hours of work increase, the marginal tax rate increases and the worker's net after-tax wage declines. The traditional way of handling this problem (Hall 1973; Wales 1973) is to use a linear approximation to the nonlinear budget constraint by, in effect, drawing a straight line, tangent to the indifference curve representing maximum attainable utility, through each individual's observed consumption bundle.

The linear approximation approach is illustrated in figure 2.3, which is a redrawing of figure 2.1 that has been corrected for the presence of progressive taxation. Three budget constraints are shown in figure 2.3. The first is the budget constraint in the absence of any income tax, which is repeated from figure 2.1. The second is the nonlinear budget constraint faced by a worker under progressive taxation. Again, this budget constraint curves toward the wage axis as the share of wages in total compensation increases. The third is the "linearized" budget constraint, which is a straight line tangent to the actual constraint at the optimal bundle of pensions and wages ($z^{**}_{w}, z^{**}_{r}$).

The linear approximation approach entails assigning price and income data to each observation as follows. First, the slope of the linearized constraint is taken as the price of wages relative to pensions ($p_{w}/p_{r}$). Second, the wage intercept of the linearized constraint ($z^{0}_{w}$ in figure 2.3) multiplied by the price of wages ($p_{w}$) is taken as total compensation. Equivalently, the observed marginal tax rate is assigned to each observation in the sample, and the sum of observed after-tax wages, pension contributions, and health insurance contributions is assigned to each observation as total compensation.

The problem posed by the linear approximation method is econo-
Figure 2.3. Progressive income taxation, the nonlinear budget constraint, and the linear approximation method.

Linearized constraint
Budget constraint, progressive income taxation
Budget constraint, no income tax

Metric: the marginal tax rate observed for each worker (or the price of wages relative to pensions based on that tax rate) is no longer exogenous under progressive income taxation. The marginal tax rate is, in effect, chosen by the worker, as is easily seen from figure 2.3. The worker depicted in the figure has chosen the compensation bundle \((z_w^{**}, z_r^{**})\), and hence faces the marginal tax rate given by the slope of the dashed line. But another worker with different preferences might choose a different compensation bundle—that is, a different point on the curved budget constraint—and would as a result face a different marginal tax rate. Hence, the marginal tax rate is chosen along with the compensation bundle, and is endogenous. In the worst case, using the observed marginal tax rate (or relative prices derived from it) as an independent variable could lead to inconsistent estimates of the effect of changes in the marginal tax rate on whatever dependent variable is of interest.
A maximum likelihood solution to the problem of nonlinear budget constraints has been suggested and implemented by Burtless and Hausman (1978) and Wales and Woodland (1979). (See also Hausman 1985; Moffitt 1986, 1990; and Megdal 1987.) For two reasons, we have not used the maximum likelihood solution to the problem posed by the nonlinear budget constraint. First, the estimation procedure used is already complex and expensive, and handling the nonlinear budget constraint by maximum likelihood would make it more so. Second, it appears that the linear approximation method we use tends to underestimate "true" elasticities of substitution estimated by maximum likelihood. Hence, even though the elasticities of substitution estimated in the next chapter tend to be large, it is perhaps best to view them as lower-bound estimates of the true elasticities of substitution between various components of compensation.

**Computation of Elasticities**

We asserted above that estimates of the parameters of the demand system would yield, in turn, estimates of the price, income, and substitution elasticities that are needed to predict the effects of various tax changes on the demand for fringe benefits. (The elasticities were written out in equations (2.8) through (2.11).)

The formulas used to compute each elasticity from the parameters of the Deaton-Muellsbauer demand system, written as equations (2.33) through (2.35), are as follows. Start with the uncompensated price elasticities (eq. 2.8). The uncompensated own-price elasticities of demand for benefit \( i \) can be computed as:

\[
\eta_{ii} = \frac{\partial z_i}{\partial p_i} \left( \frac{p_i}{z_i} \right) = \left( b_{ii} / s_i \right) - b_i - 1. \tag{2.8a}
\]

The uncompensated cross-price elasticities are:

\[
\eta_{ij} = \frac{\partial z_i}{\partial p_j} \left( \frac{p_j}{z_j} \right) = \left( b_{ij} / s_i \right) - b_i \left( s_j / s_i \right). \tag{2.8b}
\]

The compensated own- and cross-price elasticities can be computed as:
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\[ \eta_{ii}^* = \frac{\partial z_i}{\partial p_i} \left( \frac{p_i}{z_i} \right) = \frac{b_{i}}{s_i} + s_i - 1 \]  \hspace{1cm} (2.9a)

and

\[ \eta_{ij}^* = \frac{\partial z_i}{\partial p_j} \left( \frac{p_j}{z_i} \right) = \frac{b_{ij}}{s_i} + s_j. \]  \hspace{1cm} (2.9b)

Next, the income elasticities can be computed as:

\[ \eta_{im} = \frac{\partial z_i}{\partial m} \left( \frac{m}{z_i} \right) = \frac{b_i}{s_i} + 1. \]  \hspace{1cm} (2.10a)

Finally, the elasticities of substitution are:

\[ \sigma_{ii} = \eta_{ii}^* / s_i = \frac{b_{i}}{s_i^2} - \frac{1}{s_i} + 1 \]  \hspace{1cm} (2.11a)

and

\[ \sigma_{ij} = \eta_{ij}^* / s_j = \frac{b_{ij}}{s_i s_j} + 1. \]  \hspace{1cm} (2.11b)

In equations (2.8a) through (2.11b), the required parameters \((b_i, b_{ij})\) and shares \((s_i)\) are from equations (2.33) through (2.35).

Confidence Intervals for the Elasticities

Estimating the price, income, and substitution elasticities set out above is the main objective of the empirical work presented in the next chapter. Because these elasticities are central to an understanding of how tax policy changes might affect the mix of total compensation, it would be highly desirable to construct confidence intervals around each elasticity estimate, so that statistical tests of significance could be performed.

Constructing confidence intervals around the elasticities estimated using the Deaton-Muellbauer and other flexible demand systems poses a well-known problem that has been discussed by many users of flexible functional forms (Anderson and Thursby 1986; Krinsky and Robb 1986; Grant and Hamermesh 1981; Toevs 1980, 1982). It is clear from the elasticity formulas presented above that each elasticity is a nonlinear function of parameters and compensation shares. As a result, it is not obvious what measure of error should be associated with these elasticity estimates.
One possible measure of the error associated with each elasticity is the asymptotic variance (and covariances) of the estimated parameter (or parameters) underlying the elasticity in question. This is easy to calculate in cases where only one parameter is used to compute an elasticity. In other cases, this solution involves computing the variance of a linear combination of parameters. This is true, for example, in the case of the uncompensated elasticities, since both $b_i$ and either $b_{it}$ or $b_{ij}$ underlie these. 10

A remaining problem with this measure of error associated with the computed elasticities is that it treats only the parameters $(b_i, b_{it},$ and $b_{ij})$ as stochastic; the compensation shares $(s_j)$ are assumed nonstochastic. A case can be made that the shares should in fact be treated as random variables. Accordingly, an improved measure of error associated with the elasticities would be the variance of each elasticity derived by taking a Taylor expansion of each elasticity around the sample mean, which includes variances and covariances of shares, as well as variances and covariances of parameters (see, for example, Kmenta 1986, p. 486). Standard errors based on variances computed in this way are reported in tables 3.3 and 3.6 in chapter 3.

Restatement of the Model

The model of fringe benefit provision developed in this chapter is based on the following “story.” The employer offers a menu of compensation packages to workers, who select the package that maximizes their well-being. The menu offered by employers implies certain tradeoffs between components of compensation that the employer is willing to make, and these tradeoffs depend in turn on the employer’s cost of providing each benefit. The package that workers choose from the menu depends on their preferences (which depend in part on characteristics such as age and marital status), on their level of total compensation, and on the prices of the components of compensation. The prices facing workers depend not only on the employer’s costs of providing each
component of the package, but also on the differential tax treatment of each component.\textsuperscript{11}

This story can be formalized, and a model that can be estimated econometrically can be derived. We follow Deaton and Muellbauer (1980a) and specify an expenditure function in flexible form. From this flexible expenditure function, the following system of demand equations for wages, pensions, and health insurance benefits can be derived:

\begin{equation}
sw = a_w + b_{w,w}ln(p_r/p_w) + b_{w,h}ln(p_h/p_w) + b_wln(m/P*) + d_w1x_1 + \ldots + d_wKx_K + u_w
\end{equation}

\begin{equation}
sr = a_r + b_{r,r}ln(p_r/p_w) + b_{r,h}ln(p_h/p_w) + b_rln(m/P*) + d_r1x_1 + \ldots + d_rKx_K + u_r
\end{equation}

\begin{equation}
s_h = a_h + b_{h,r}ln(p_r/p_w) + b_{h,h}ln(p_h/p_w) + b_hln(m/P*) + d_h1x_1 + \ldots + d_hKx_K + u_h.
\end{equation}

This is a standard set of demand equations, in the sense that the demand for each component of compensation is modeled as a function of the prices of those benefits, income, and other characteristics such as age and sex. In these equations, \( s_w, s_r, \) and \( s_h \) are the shares (or proportions) of total compensation received in the form of wages, pension contributions, and health insurance benefits; \( (p_r/p_w) \) and \( (p_h/p_w) \) are the prices of pensions and health insurance, relative to wages, that face workers; \( m \) is total compensation in dollars; \( P^* \) is a price index approximated by \( ln P^* = s_wln p_w + s_rln p_r + s_hln p_h \); \( x_1 \) through \( x_K \) are control variables other than prices and income, such as demographic characteristics, that might affect the demand for fringe benefits; and \( u_w, u_r, \) and \( u_h, \) are random disturbance terms that are assumed to be normally distributed with zero mean.

The \( a_i, b_{ij}, b_i, \) and \( d_{ik} \) are parameters to be estimated. These parameters can be interpreted as follows. The \( b_{ij} \) parameters show the effect of changes in relative prices on the budget shares, holding real income constant. More precisely, a 1 percent increase in the price of component \( i \) changes the share of component \( j \) by \( (b_{ij}/100) \), other things equal. The \( b_i \) parameters show the effect of changes in real total compensation on compensation shares. For a component of compensation that is a luxury,
$b_i$ will be positive; for a component that is a necessity, $b_i$ will be negative. In addition, estimates of the $b_{ij}$ and $b_i$ parameters yield in turn estimates of the price, income, and substitution elasticities that are needed to determine the effect of changing tax policy on the provision of the different forms of fringe benefits. Finally, the $d_{ik}$ show the influence of the demographic and other characteristics on each compensation share. A unit increase in any of these other control variables will change the compensation share in question by $d_{ik}$, all else equal.

Two final points deserve emphasis. First, the relative prices in the demand system are constructed with an eye to both the employer's cost of providing each component of compensation, and to the tax treatment of each component. Specifically, the relative prices are defined by:

$$\frac{p_w}{p_r} = \frac{c_w}{c_r} \left(1 - t\right)$$

(2.5)

and

$$\frac{p_w}{p_h} = \frac{c_w}{c_h} \left(1 - t\right)$$

(2.36)

where $c_w$, $c_r$, and $c_h$ are the employer's cost of providing a unit of pension benefits, a unit of wage benefits, and a unit of health insurance benefits; and $t$ is the marginal tax on income faced by the worker. Actual measurement of these relative prices is an important part of the work presented in the next chapter. Second, earlier work on fringe benefits has examined only the choice between wages and fringe benefits taken as a whole. The demand system set out above specifies separate equations for pensions and health insurance, and hence allows examination of tradeoffs within the fringe benefit package. It is necessary to estimate only two of the three equations (2.33) through (2.35), because only two are independent. Although the choice of which equations to estimate is arbitrary, we estimate the two fringe benefit share equations [(2.34) and (2.35)] in the next chapter, because our main concern is with the influence of changing prices and incomes on fringe benefits.
Appendix to Chapter 2
Summary of Notation

$a'_i, a_i$ Share equation intercept terms in the Deaton-Muellbauer Almost Ideal Demand System. Note that $a_i = a'_i - b_i (\ln g)$. See below for definitions of $b_i$ and $g$.

$b_i$ Parameters, showing the effect of changes in real total compensation on compensation shares, in the Deaton-Muellbauer Almost Ideal Demand System.

$b'_{ij}, b_{ij}$ Parameters showing the influence of changes in relative prices on compensation shares in the Deaton-Muellbauer Almost Ideal Demand System. Note that $b_{ij} = (\frac{1}{2})(b'_{ij} + b''_{ij})$.

$c_w, c_r, c_h$ Cost to the employer of a unit of wage benefits, pension benefits, and health insurance benefits.

$C(.)$ The consumer cost or expenditure function, indicating the minimum expenditure needed to attain a given level of utility $(U)$ at a given set of benefit prices.

$d_{ik}$ Parameters showing the influence of the control variables $(x_1, \ldots, x_K)$ on compensation shares in the Deaton-Muellbauer Almost Ideal Demand System.

$g$ Scalar indicating the closeness of the approximation $P^*$ to the true price index $P$.

$m$ Total compensation in dollars, exogenously set by the employer.

$m'$ Instrument for real total compensation used with the EEEC data in chapter 3. Equals average after-tax earnings of each worker group divided by the industry average share of compensation received as wages.

$m_1$ Total compensation paid in wage and pension benefits (health insurance benefits predetermined).
Tax Treatment of Fringe Benefits

$m_2, m_3$ Total compensation paid in pension and health insurance benefits (wage benefits predetermined).

$p_w, p_r, p_h$ Prices faced by workers of wage benefits, retirement benefits (referred to as pensions throughout the text), and health insurance benefits.

$P$ Aggregate price index of benefits.

$P^*$ Approximation to the aggregate price index used in the empirical work.

$s_i$ The share (proportion) of total compensation received as benefit $i$.

$t$ The marginal tax rate on income faced by the worker.

$u_w, u_r, u_h$ Random disturbance terms in the wage share, pension share, and health insurance share equations.

$U(.)$ The direct utility function, indicating the utility derived from consumption of a given bundle of benefit quantities.

$V(.)$ The indirect utility function, indicating the maximum utility attainable at a given set of benefit prices, and a given level of total compensation.

$x_1, \ldots, x_K$ Demographic, industry, and other control variables (other than prices and real total compensation) included in the benefit demand functions.

$z_w, z_r, z_h$ Quantities of wage benefits, retirement benefits (referred to as pensions throughout the text), and health insurance benefits. Optimal quantities are shown with asterisks.

$\eta_{ii}, \eta_{ij}$ The uncompensated own- and cross-price elasticities of demand for benefit $i$.

$\eta_{im}$ The income (or expenditure) elasticity of demand for benefit $i$. 
The compensated own- and cross-price elasticities of demand for benefit $i$.

The elasticity of substitution between benefit $i$ and benefit $j$. A positive $\sigma_{ij}$ indicates that $i$ and $j$ are substitutes, a negative $\sigma_{ij}$ indicates that they are complements.

NOTES

1 It is true that employers may offer several types of health benefits and that pension benefits also may be somewhat flexible. Regarding health benefits, the variation may be in the delivery system—health maintenance organization, fee-for-service provider, or independent practice association. In some cases, the actual premiums charged will differ, as will the service provided. Hence, employees are allowed to choose, in some cases, from a limited menu of benefit packages offered by a given employer, in addition to having different menus offered by different employers.

2 One can easily imagine a model of benefit determination that views the firm as providing a package of benefits that minimizes the firm's cost of retaining a workforce of given size and quality. But in such an approach, workers' responses to the tax treatment of benefits would be confounded with their underlying preferences for quantities of benefits.

3 The limited experience rating of unemployment insurance and workers' compensation implies that firms do have some control over these legally mandated expenses. The difficulty economists have had in estimating a behavioral response of firms to experience rating, however, suggests that these expenses are largely beyond a firm's control.

4 As already noted, we examine wages, pensions, and health insurance because they are the three largest components of total compensation that are provided voluntarily. Again, we omit fringe benefits that employers must provide by law—social security, unemployment insurance, and workers' compensation contributions, for example—because workers and employers have limited scope for choosing these. By omitting these so-called mandatory fringe benefits, the utility function (2.1) embodies the assumption that wages and voluntarily provided fringe benefits are (as a group) weakly separable from all other goods that provide utility to workers. This assumption of weak separability implies that changes in the consumption of goods other than wages, pensions, and health benefits do not affect the marginal rate of substitution between, for example, wages and pensions. It would be useful to test empirically the assumption of separability, although no straightforward way of doing so suggests itself.

5 The same points could be illustrated by fixing pension benefits and analyzing substitution between wages and health insurance, since for now we are assuming that wages are taxed and all fringe benefits are untaxed.
Presumably, the two employers being considered would operate in different product and labor markets. Accordingly, total expenditures on pensions and health insurance could differ between the two employers without violating competitive assumptions. From an empirical standpoint, the key is to include appropriate controls for industry and workers' characteristics.

Our approach is open to the criticism that pension provision and specific human capital are jointly determined; that is, causation does not run from specific human capital to pension provision, as our empirical specification would suggest.

These are the results found by Wales and Woodland (1979). It is not obvious that their results can be generalized, but their case is similar to ours.

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Note that formulas (2.8a) through (2.11b) apply when the approximation to the price index \( P^* \) is used. Use of the approximation \( P^* \) results in elasticity formulas for the Deaton-Muellbauer demand system that differ from those that use the exact price index \( P \) — see, for example, Anderson and Blundell (1983).

Consider for example, the variance of \( \eta_{ij} \):

\[
\text{var} (\eta_{ij}) = \text{var} \left[ \frac{1}{s_i}b_{ij} - \frac{s_j}{s_i}b_j \right] \\
= \left[ \frac{1}{s_i^2} \text{var}(b_{ij}) + \frac{(s_j/s_i)^2 \text{var}(b_j)}{s_i} \right] \\
- \frac{2b_j(s_j/s_i)^2 \text{cov}(b_{ij}, b_j)}{s_i}.
\]

Prices facing workers may in addition depend on the competitive position of the employer. That is, employers who possess market power may obtain rents by providing their employees with benefits obtained at favorable rates. We are grateful to William Alpert for this point.

If \( \eta_{im} \) exceeds unity, then good \( i \) is a luxury, otherwise the good is a necessity. Referring to equation (2.10a), \( \eta_{im} = (b_i/s_i) + 1 \). Hence, \( \eta_{im} \) will always exceed unity if \( b_i \) is positive, but cannot exceed unity if \( b_i \) is negative. Also, if \( \eta_{im} \) is greater than zero, the good in question is normal; if \( \eta_{im} \) is negative it is inferior. To determine whether a component of compensation is normal or inferior, one needs to use (2.10a); in general, though, \( b_i \) must be negative and \( s_i \) must be small in order for \( \eta_{im} \) to be inferior.

Note again that the price ratios in equations (2.5) and (2.36) define the inverses of those used in the estimating equations (2.33) through (2.35).
The TAX Treatment of Fringe Benefits

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